

Gravity currents as a test case: do simple mathematical models produce good physical insights, or do good insights generate powerful models?

Prof. Marius Ungarish
Department of Computer Science, Technion, Haifa, Israel

Abstract

A gravity current appears when fluid of one density, ρ_c , propagates into another fluid of a different density, ρ_a , and the motion is mainly in the horizontal direction. A gravity current is formed when we open the door of a heated house and cold air from outside flows over the floor into the less dense warm air inside. A gravity current is formed when we pour honey on a pancake and we let it spread out on its own. Gravity currents originate in many natural and industrial circumstances and are present in the atmosphere, lakes and oceans as winds, cold or warm streams or currents, polluted discharges, etc. A gravity current which propagates inside a stratified fluid (rather than along a boundary) is called “intrusion”.

Simple qualitative consideration and observations indicate that the gravity current is a very complex, multi-faced, and parameter-rich physical manifestation. Nevertheless, the gravity current also turns out to be a modeling-friendly phenomenon. Indeed, visualizations of the real flow field reveal an extremely complicated three-dimensional motion, with an irregular interface, billows, mixing, and instabilities. The accurate numerical simulation of this flow from the full set of governing equations (the Navier-Stokes system) requires weeks of number-crunching on powerful computer arrays. On the other hand, there are “mathematical models” based on a long line of assumptions such as hydrostatic pressure, sharp interface, Boussinesq system, thin layer, idealized release conditions. This simplified set of equations enables us to determine the behavior of the averaged variables entirely from analytical considerations and/or numerical solutions that require insignificant CPU time.

The lecture gives a brief presentation of some typical models and solutions. We see that: (a) Qualitatively, the simplified theory is able to provide the governing dimensionless parameters and the salient features of the various flow regimes; and (b) Quantitatively, the simple models predict velocities of propagation which agree with experiments and full Navier-Stokes simulations within a few percent, sometimes within the range of the experimental errors. The fact that such simple models give useful results merits attention: is the predictive power of the model a consequence of postulated mathematical simplicity, or is it rather a result of well-selected physical components? Some tentative answers to this universally relevant question, as provided by the test-case of gravity currents and intrusions, are discussed.

References

- [1] M. Ungarish. *An Introduction to Gravity Currents and Intrusions*. Chapman & Hall/CRC press, Boca Raton London New York, 2009.