Linear$^+$ modelling of turbulent shear flow

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Organised motion in turbulent shear flow, first observed in the 1960s [1], motivates the search for an underlying dynamic law simpler than that expressed by the Navier-Stokes system and that might clarify the mechanisms that underpin turbulence. The first such models, proposed in 1970s, are based on linearisation about the turbulent mean [2, 3]. They have since been found to describe many characteristics of coherent structures in high-Reynolds-number turbulent jets [4], and they have been used to clarify the mechanisms by which turbulent jets generate sound [5].

Mean-flow-based linearised models have recently been revisited, formulated with the non-linear equations split into a linear operator and an endogenous forcing term associated with non-linear scale interactions (NLI). Written in input-output form, NLI and state can thus be related by the resolvent of the linear operator. This acoustic-analogy-like [7, 8, 9] formulation provides a surprisingly versatile framework for problems involving high-Reynolds-number turbulent shear flow; particularly so when, as first done by Sharma & McKeon [11] and Hwang & Cossu [12], a singular-value decomposition of the mean-flow-based resolvent operator is considered. This has opened the door to the development of many novel strategies for analysis, modelling, estimation and control. Resolvent analysis has been used for the study of compressible turbulent jets by Towne *et al.* [13], Lesshafft *et al.* [14], Cavalieri *et al.* [21], Nogueira *et al.* [15], Pickering *et al.* [16], and Maia *et al.* [17]; for the study wall-bounded flows [18, 22], wakes [19], jet-noise [6, 23] and trailing-edge noise [20], to mention just a handful of applications.

The talk will overview a number of recent numerical and experimental studies where resolvent-based models have been used: (1) for the exploration of self-similarity in wall-bounded flows [22]; (2) for the analysis and modelling of sound-generation mechanisms in turbulent jets [23]; (3) for the estimation and control of free and wall-bounded turbulent shear flows [24, 25, 26, 27, 28]. Our objective will be to highlight the surprising versatility of mean-flow-based linear models, but also to discuss their limitations.

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