Conditions for local generation of NonLinear Internal Waves in a pycnocline: a numerical study.

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The “local” NLIW generation

New & Pingree 1990: in the Bay of Biscay (BoB), solitary waves arise (too) far from the continental slope

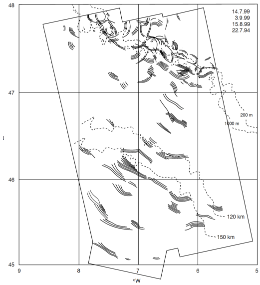


Figure: SAR images of the BoB gathered (New & Da Silva 2002)

Explanation (New & Pingree 1990): “local” generation, with an IW beam impinging the thermocline.

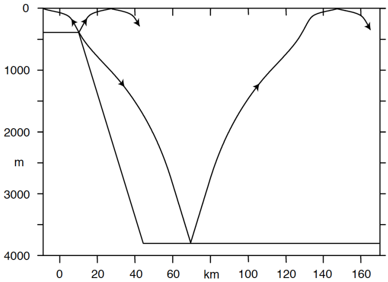


Figure: Ray paths in the BoB near 47°N (New & Da Silva 2002)

Objectives of the present study

What we have now:

- observations in the ocean: BoB, Konyaev 1995 (Indian ocean), New & Da Silva 2002 (remote sensing), Da Silva *et al.* 2007 (Portugal), Da Silva *et al.* 2008 (Mozambique Channel),
- a few theoretical models (Thorpe 1998, Gerkema 2001, Akylas *et al.* 2007, Maugé & Gerkema 2007).

What is missing:

- direct observations of the generation of solitary waves, whether in the oceans, in the lab or using DNS.

What we will present:

- DNS using an idealized setup,
- simple rules to understand the selection of the mode of the NLIWS.

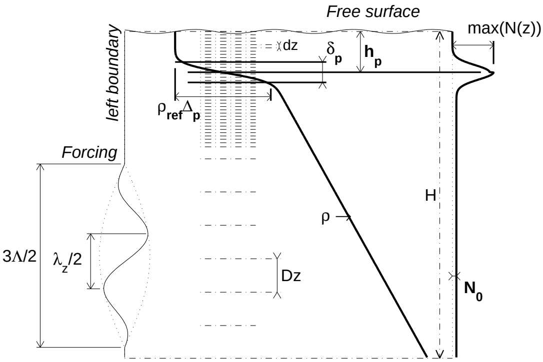
The set-up

Total depth H, stratification:

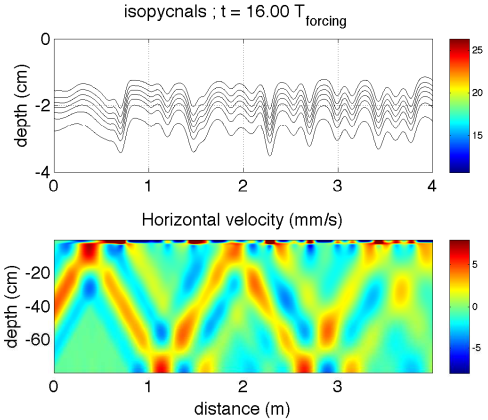
1. upper layer, $N = 0$, $h_p/H = 1/40$,
2. pycnocline, $\delta_p/H = 1/80$, $\Delta\rho/\rho_{ref} = \Delta_p$
3. lower layer, constant N_0.

Forcing of the velocity on the left boundary, vertical scale Λ.

Code: MITgcm.



Generation of mode-1 NLIWs



Experiment E1: $\Delta_p = 2.05$ %, $\Lambda/H = 0.75 \Rightarrow$ **mode-1 NLIWs**.

The modal analysis

Taylor-Goldstein equation with Ω (frequency of the interfacial displacement), illustration for E1:

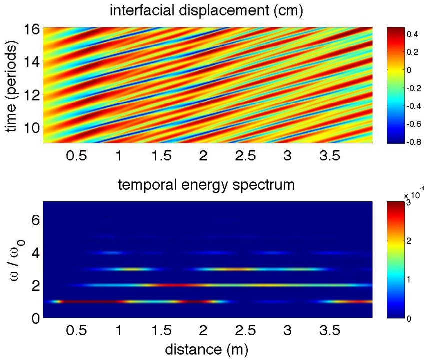
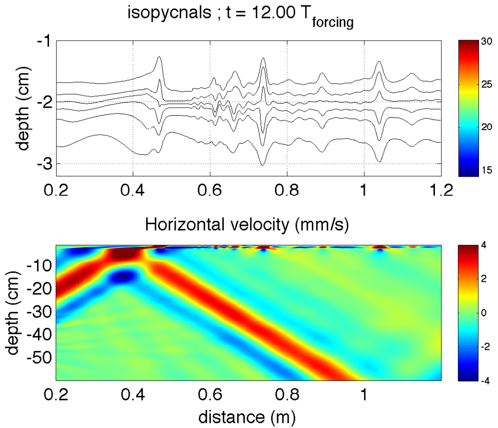


Figure: Inverted Hovmöller diagram & corresponding periodograms

- $\Omega \approx 3 \times \omega_{forcing}$
- phase speed of the 1^{st} mode $c_1(\Omega)$ compared to the horizontal phase speed of the IW beam:

$$\frac{\mathbf{c_1}}{\mathbf{v_{beam}}} \simeq \mathbf{1.3}$$

Application to mode-2 NLIWs

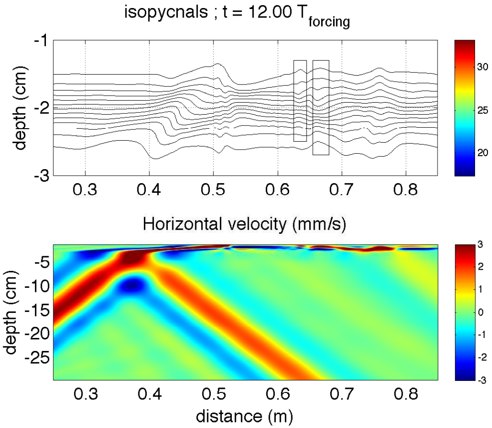


Experiment E2: $\Delta_p = 3.38$ %,
$\Lambda/H = 0.325 \Rightarrow$ intermediate case

- $\Rightarrow$ **mode-2 NLIWs**, $\Omega \approx 3\omega_{forcing}$
- phase speed of the 2^{nd} mode $c_2(\Omega)$ compared to the horizontal phase speed of the IW beam:

$$\frac{\mathbf{c_2}}{\mathbf{v_{beam}}} \simeq \mathbf{0.94}$$

Application to mode-3 NLIWs



Experiment E3: $\Delta_p = 4\ \%$, $\Lambda/H = 0.188 \Rightarrow$ "fast" interfacial modes, "slow" IWB

- $\Rightarrow$ **mode-3 NLIWs**, $\Omega \approx 2\omega_{forcing}$
- phase speed of the 3^{rd} mode $c_3(\Omega)$ compared to the horizontal phase speed of the IW beam:

$$\frac{\mathbf{c_3}}{\mathbf{v_{beam}}} \simeq \mathbf{1.06}$$

Summary: a condition for the generation of mode-n NLIWs

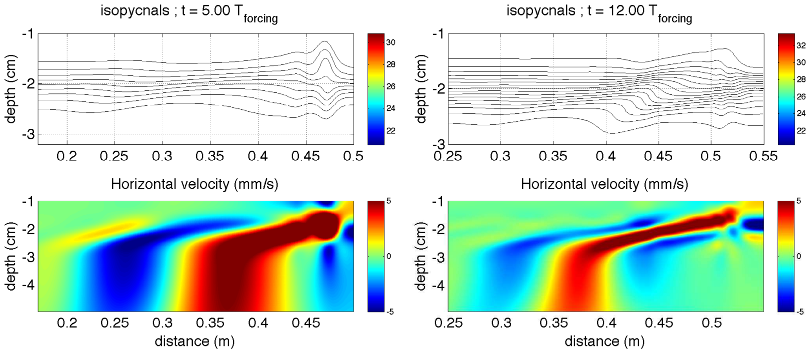
To generate mode-n NLIWs of frequency Ω in the pycnocline, make sure that these two match:

- horizontal phase speed of the IW beam of frequency $\omega_{forcing}$,
- horizontal phase speed of the mode-n IW of frequency Ω ($> \omega_{forcing}$).

What happens at the beginning of the generation?

Close observation of the impact zone

Let us have a closer look at the pycnocline:



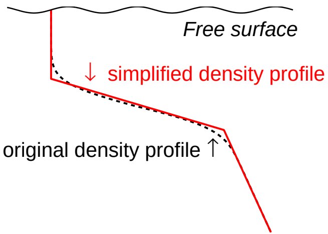
(a) E2 (b) E3

Refraction of the beam in the pycnocline

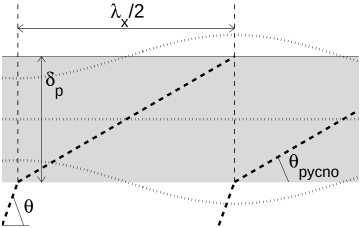
A simplified view on the problem

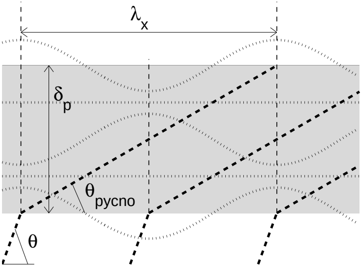
Three approximations will be made:

- beam $\Rightarrow$ plane wave
- linear dynamics (early stages of the generation)
- three-layer model, conservation of $\int N^2(z)dz$:



Resonance condition for mode-n ($n \geq 2$) NLIWs





Mode-2 NLIWs:
$\lambda_x \tan\left(\theta_{pycno}\right) = \lambda_z^{pycno} = 2\delta_p$

Mode-3 NLIWs:
$\lambda_x \tan\theta_{pycno} = \lambda_z^{pycno} = \delta_p$

In E2: $\dfrac{\boldsymbol{\lambda}_\mathbf{z}^\mathbf{pycno}}{\mathbf{2\delta_p}} \simeq \mathbf{0.88}$

In E3: $\dfrac{\boldsymbol{\lambda}_\mathbf{z}^\mathbf{pycno}}{\boldsymbol{\delta}_\mathbf{p}} \simeq \mathbf{0.94}$

To generate mode-n NLIWs, make sure that:

$$\frac{\mathbf{n-1}}{\mathbf{2}} \times \frac{\lambda_{\mathbf{z}}^{\mathbf{pycno}}}{\delta_{\mathbf{p}}} = \mathbf{1} \qquad (n \geq 2)$$

The vertical structure of the IW field in the pycnocline has to be similar to a confined mode-n IW (Bragg-like resonance condition).

Summary

Two different conditions for the generation of mode-n NLIWs.

- **Condition generation using a modal analysis:** if the ϕ-speed of the IWB match the ϕ-speed of the mode-n internal wave of frequency Ω trapped in the pycnocline, mode-n NLIWs of frequency $\simeq \Omega$ can be generated.
- **Bragg-like resonance condition:** given the strength and thickness of the pycnocline, if the incident beam has such a wavelength that it triggers a mode-n structure in the pycnocline, mode-n NLIWs can be generated.
- Are these two independent conditions compatible ?
- In both models, what happens between the impact on the pycnocline and the fully developed NLIWs ?
- Needs further checking from the ocean (ongoing), experiments (ongoing), other numerical works. . .

Bonus: comparison of E1 with previous theoretical models

- **Gerkema 2007:**

$$\gamma = \frac{\sqrt{g\Delta_p h_p}}{N_0 H} \left(= \frac{\text{pure interfacial waves}}{1^{st}\text{ internal mode without pycnocline}} \right).$$

Here : $\gamma \simeq 0.132$ (instead of $\gamma \simeq 0.12$ in 2001's paper).

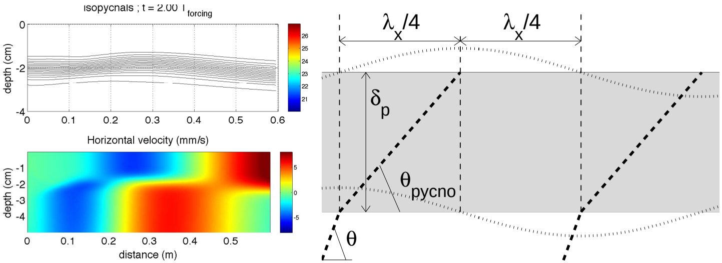
- **Akylas et al. 2007:**

$$\alpha = \frac{N_0 \Lambda}{\sqrt{g\Delta_p h_p}} \left(= \frac{\text{horizontal } \phi\text{−speed of IWB}}{\text{pure interfacial waves}} \right).$$

Here : $\alpha \simeq 4.07$ (instead of $\alpha \simeq 6.28$ in 2007's paper).

Bonus: resonance condition of mode-1 NLIWs

μ_1 isn't defined yet. A ϕ-shift can still be seen in E1:



$\lambda_x \tan\theta_i = 4\delta_p$? That would extend the definition of μ_n:

$$\mu_n = \frac{(n - 1 + \delta_{1n}/2)\lambda_x \tan\theta_i}{2\delta_p} = 1 \qquad (\delta_{1n} = 1 \Leftrightarrow n = 1)$$