SEAWATER MAGNETOHYDRODYNAMIC THRUSTER: MODEL AND UPSCALING

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Abstract: Naval magneto-hydrodynamic (MHD) propulsion's challenge relies on the high level of Joule dissipation associated to the low electrical conductivity of seawater. To reduce it, a strong flowrate should be exposed to the Lorentz force, but then the hydraulic loss will increase. This paper presents a simple analytical model to estimate the Joule dissipation and hydraulic losses in a straight MHD channel, producing its dimensionless performance (head and pump efficiency) as a function of three dimensionless parameters (current, velocity, loss coefficient). Adding some other coefficients (additional loss coefficient, ratio of the 'drag front section' to the channel cross section, aspect ratios of the channel) makes it possible to estimate the cruise speed of a boat equipped with such a channel working as a MHD thruster.

Keywords: MHD pump, MHD propulsion, model boat.

1. Introduction. Magneto-Hydro-Dynamic (MHD) thrusters for naval applications have been developed intensively some decades ago, culminating with the 30-m Yamato boat that reached a speed of 5.5 knots with 4-T superconducting magnets [1]. Recent high-temperature superconducting ribbons may broaden the possibilities if they are integrated onboard in large magnets providing strong magnetic fields in straight channels. This study aims at precising how large the channels and how strong the magnetic field should be, using a simple hydraulic model for the thruster, considered as a pump, and a fixed drag coefficient of the boat (without the drag added by the channels) to evaluate the velocity reached for a given power supply.

To evaluate this approach, a model boat (Fig 1) was built using permanent magnets in place of superconducting ones, and tested in a pool with two 22.2 V batteries in series as an electric supply. The terminal speed of the boat reached 0.3 m/s in artificial seawater. The bubbles produced by electrolysis are correctly evacuated by the flow even for a current density of 7 kA/m² on the electrodes. Two divergent sections have been tested at the channel outlet, the longer one reducing the boat speed to 0.2 m/s.

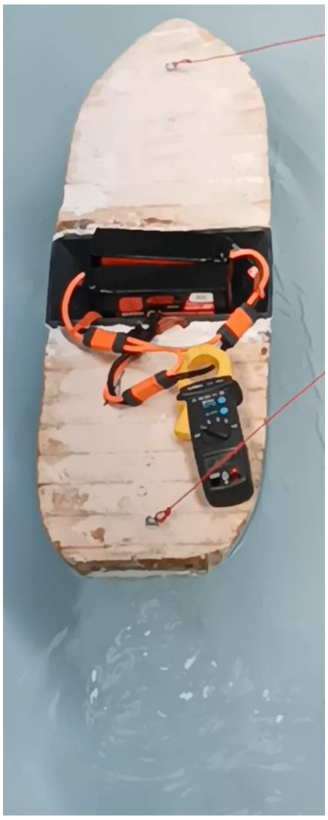

Figure 1. Model boat in test

This paper presents a simple analytical model of the MHD channel, considered as a pump, with emphasis on dimensional analysis and up-scaling. The cruise speed of the boat is also evaluated using dimensionless parameters such as its drag coefficient and a loss factor inside the propulsion channel.

2. Model for the MHD channel. The central part of the MHD thrusters considered here is the MHD channel guiding a fluid flow at velocity $\boldsymbol{U}$, inserted in a magnet providing a cross flow magnetic field $\boldsymbol{B}$ and equipped with electrodes forcing a current density $\boldsymbol{j}$ almost perpendicular to $\boldsymbol{U}$ and $\boldsymbol{B}$. For the sake of simplicity we consider here that those vector fields are perfectly perpendicular and uniform in a brick-shaped domain (Fig 2), of dimensions L_U, L_B and L_j along each vector field. Such a channel is also the central part of a MHD pump.

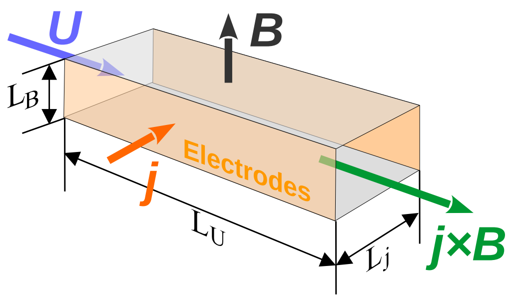

Figure 2. Geometry of the MHD channel

The Ohm's law can be written $j=\sigma(\Delta\varphi/L_j-UB)$ where $\sigma\approx 5\ \Omega^{-1}\text{m}^{-1}$ is the conductivity of seawater and $\Delta\varphi$ is the potential difference imposed to the solution by the electrodes. However, the anode (resp. cathode) is at a potential $E_a>0$ (resp. $E_c<0$) with respect to the solution nearby, because of the electrolysis needed to have current flowing into a ionic solution. Therefore the voltage difference between the electrodes, $\Delta\varphi_{el}=\Delta\varphi+(E_a-E_c)$, contains an electrochemical term E_a-E_c estimated from 2 V to 6 V when j increases between 0 and 1 kA/m² [2]. Using the total current $I=jL_UL_B$, we obtain the total electrical power P_{el}, its part P_{chim} consumed by electrochemical reactions and the rest $P_{channel}$ tranferred to the solution:

$$P_{el}=I.\Delta\varphi_{el} \quad ; \quad P_{chim}=I.(E_a-E_c) \quad ; \quad P_{channel}=I.\Delta\varphi \tag{1}$$

Notice that the ratio $\eta_{chim}=P_{channel}/P_{el}$ is usualy greater than 0.9 or higher for large size channels.

Integrating the Ohm's law over L_U and L_B, we find that $\Delta\varphi=RI+UBL_j$, where $R=L_j/(\sigma L_UL_B)$ is the electrical resistance of the channel and UBL_j the counter-electromotive force due to the fluid velocity. Multiplying by I, we find the Joule dissipation RI^2, i.e. the integral of j^2/σ, and the integral of f_LU (where $f_L=jB$ is the Lorentz force), i.e. the mechanical power transferred to the fluid:

$$\Delta\varphi=RI+UBL_j \Rightarrow P_{channel}=P_{joule}+P_{mhd} \quad ; \quad P_{joule}=RI^2 \quad ; \quad P_{mhd}=IBUL_j \tag{2}$$

Of course P_{joule} is lost for propulsion or pumping, and maximizing the MHD efficiency $\eta_{mhd}=P_{mhd}/P_{channel}$ is a keypoint of any design. As for any electric receiver, $\eta_{mhd}=UB/(\Delta\varphi/L_j)$ increases when the applied voltage $\Delta\varphi$ decreases towards the counter-electromotive force UBL_j, but then the current vanishes and the thrust or pump head vanishes also.

As in any hydraulic pump, the mechanical power P_{mhd} is partly dissipated by friction inside the pump, and the external circuit may only use the remaining part, i.e. the hydraulic power $P_{hyd}=Q\Delta p_{t,pump}$, convected out of the pump by the flowrate $Q=UL_BL_j$ flowing in the pump. Here $\Delta p_{t,pump}$ is the total pressure (or head) supplied by the pump (mechanical energy per unit volume of fluid). This head $\Delta p_{t,pump}$ corresponds to a pressure rise in a straight channel as the one of Fig 2, but it may also include a variation of the kinetic energy per unit volume, $\frac{1}{2}\rho U^2$, if the section evolves along the channel. The head loss $\Delta p_{t,loss}$ is expressed using a head loss coefficient k_{loss} (considered constant here), and P_{mhd} is expressed from Eq. 2, so that:

$$P_{mhd}=P_{loss}+P_{hyd} \ ; \ P_{hyd}=Q\Delta p_{t,pump} \quad \Rightarrow \quad \Delta p_{t,pump}=(B/L_B)I-(k_{loss}/2)\rho U^2 \tag{3}$$

Notice that k_{loss} is an important parameter, containing regular losses fL_U/D_h where $D_h=4L_BL_J/(2L_B+2L_J)$ is the hydraulic diameter and f is the friction factor, but also minor losses due to entry effects of to geometric imperfections. The friction factor depends on the wall rugosity divided by D_h, and possibly on the velocity except at very high Reynolds numbers. The hydraulic efficiency $\eta_{hyd}=P_{hyd}/P_{mhd}$ is therefore difficult to predict.

Eq. 3 represents the characteristic of the pump for a given current I. A dimensionless equivalent can be built using reference values based on B, σ, ρ, L_U, i.e. the fixed design parameters. The dimensionless velocity $U_a=\rho U/(\sigma B^2 L_U)$ is the inverse of a Stuart number, and is also a dimensionless flowrate: $U_a=\rho Q/(\sigma B^2 L_U L_B L_J)$. The dimensionless current density (and dimensionless current) is $I_a=\rho j/(\sigma^2 B^3 L_U)=\rho I/(\sigma^2 B^3 L_U{}^2 L_B)$. Eq. 3 becomes:

$$\Delta p_{ta}=I_a-½k_{loss}U_a^2 \quad \text{where} \quad U_a=\frac{\rho U}{\sigma B^2 L_U}\ ,\ \Delta p_{t,a}=\frac{\rho\,\Delta p_{t,pump}}{\sigma^2 B^4 L_U^2}\ ,\ I_a=\frac{\rho I}{\sigma^2 B^3 L_U^2 L_B} \tag{4}$$

The pump efficiency $\eta_{pump}=\eta_{mhd}\eta_{hyd}=P_{hyd}/P_{channel}$ may also be expressed using U_a and I_a:

$$\eta_{pump}=\frac{U_a}{I_a}\frac{I_a-(k_{loss}/2)U_a^2}{I_a+U_a} \tag{5}$$

Eq. 4 and Eq. 5 are plotted in Fig 3 for a loss coefficient k_{loss}=1 representative of the MHD pump and model boat studied experimentally in our team (see below). A lower value of k_{loss} would, for each value of I_a, enlarge the range of flowrate available before the head drops, and increase the maximum efficiency, that is reached just before this dropout.

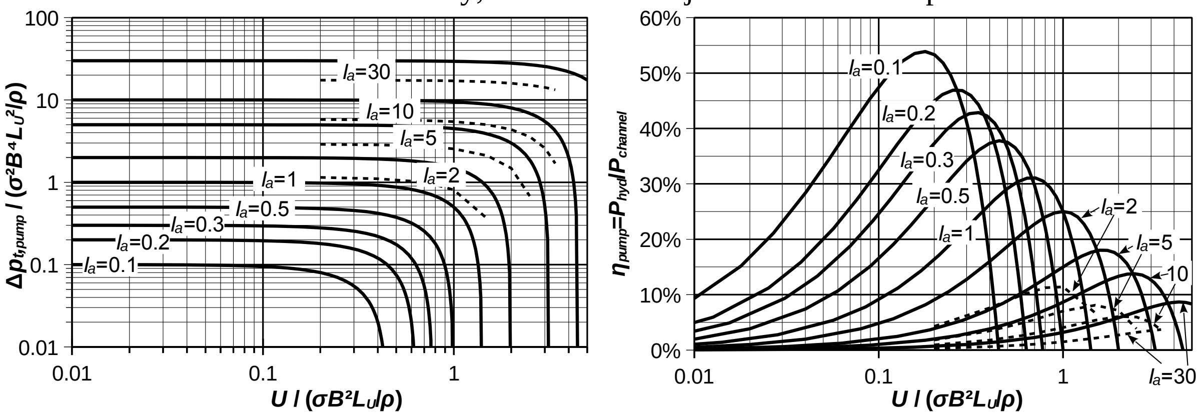

Figure 3. Dimensionless characteristics for k_{loss}=1. Dashed lines for the pump experiment (see below)

3. Model of the boat. A MHD boat (sketched in Fig 4) is equiped with a propulsion channel comprising a MHD pump (see Fig 2), an inlet section, often converging, and an outlet section that may be slightly diverging (with the risk of detachement and high head loss). The figure also shows the flow tube passing through the thruster, and its upstream section where the fluid velocity V_∞ is equal to the boat speed, and the relative modified pressure p^*_∞ is 0. The areas A_i and A_o of the inlet and outlet sections are known from their ratios $\alpha_i=A_i/A_{mhd}$ and $\alpha_o=A_o/A_{mhd}$ to the crossflow section $A_{mhd}=L_B L_J$ of the MHD channel. We consider here that p^*_o=0 since the jet is in equilibrium with the external water flow, which is similar to a wake flow in this region. Then a momentum balance on the propeller (section between A_i and A_o) gives the thrust:

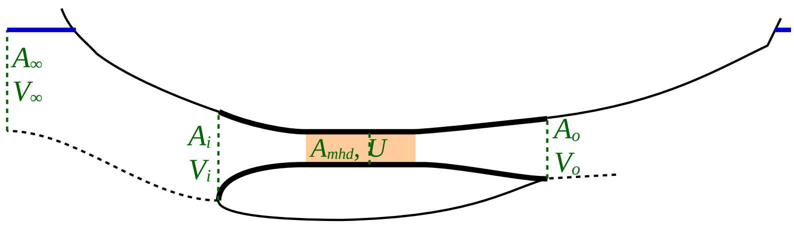

Figure 4. Propulsion channel of a MHD boat

$$T=\left[\rho U^2\left(\frac{1}{\alpha_o}-\frac{1}{\alpha_i}\right)-p_i^*\alpha_i\right]A_{mhd} \tag{6}$$

The thrust in Eq. 6 includes the reaction to the MHD force applied to water, but also the pressure and friction forces applied on the internal walls of the thruster (bolded lines on the figure), generally downstream (drag) and balancing an important part of the MHD force.

When the boat speed is stable (cruise conditions), the thrust equilibrates the drag of the boat (excluding the inner drag of the channel), expressed here using a constant drag coefficient C_D and the projected frontal area S of the boat. We suppose that the inlet is adapted to the cruise velocity, i.e. $p_i^*=0$. This is obtained only for $\alpha_i=(U/V_\infty)_{cruise}$, other values will decrease the thrust but only weakly. The thrust to drag equilibrium becomes:

$$\rho U^2 A_{mhd}\left(\frac{1}{\alpha_o}-\frac{V_\infty}{U}\right)=S\,C_D\frac{\rho V_\infty^2}{2} \;\Rightarrow\; \frac{1}{\alpha_o}\left(\frac{U}{V_\infty}\right)^2-\frac{U}{V_\infty}=\frac{\alpha_{SCD}}{2} \;\Rightarrow\; \frac{U}{V_\infty}=\frac{\alpha_o}{2}\left(1+\sqrt{1+2\frac{\alpha_{SCD}}{\alpha_o}}\right) \quad (7)$$

where $\alpha_{SCD}=S.C_D/A_{mhd}$ is a dimensionless drag section, increasing with the drag coefficient and decreasing when the part of the front section occupied by the thruster increases.

The channel velocity is given as a function of the pump regime (i.e. of IB), by the generalized Bernoulli equation from A_∞ to A_o. Eliminating U between that equation and Eq. 7, we obtain the boat cruise speed as a function of the pump regime:

$$\frac{\rho}{2}\left(\frac{U}{\alpha_o}\right)^2=\frac{\rho V_\infty^2}{2}-k_{tot}\frac{\rho U^2}{2}+\frac{I\,B}{L_B} \;\Rightarrow\; \frac{\rho V_\infty^2}{2}=\frac{4\,I\,B/L_B}{\left(1+\alpha_o^2 k_{tot}\right)\left(1+\sqrt{1+2\frac{\alpha_{SCD}}{\alpha_o}}\right)^2-4} \quad (8)$$

Notice that the loss coefficient k_{tot} includes the one of the pump (k_{loss}, see Eq. 3), and additional parts due to the entry section and exit nozzle. Our analytical model is based on uniform profiles in each section, so that it is not suited to describe a divergent with strong recirculations (even with an increased loss coefficient k_{loss}).

4. Applications of the models. The main parameters of the pump experiment [3] and the model boat developped in our laboratory are exposed in Table 1:

Table 1. Parameters of the experiments

	L_U / mm	L_B / mm	L_J / mm	σ / (Ω⁻¹m⁻¹)	ρ / (kg.m⁻³)	B / T	S / m²
Pump exp.	3200	39	80	19.5	1163	3.51;4.09;4.68	-
Model boat	100	28	32.5	5.26	1025	0.57	0.0163

The salinity of water was 35 g/l for the boat and 200 g/l for the pump (to increase σ). The values of B in the table are computed averages (on the MHD channel) of the component along L_B, and the three values for the pump correspond to B_{ctr}=3 T;3.5 T;4 T at the center of the magnet. The front section of the boat takes into account the real immersion of the hull.

For the pump (that has a 3-turns helical channel in place of a straight one as in Fig 2), we measured experimentally the channel voltage $\Delta\varphi$ and the pressure gain Δp_{t61} produced by 2.5 turns of the helix. All experiments (for 30 A<I<300 A and 2.5 m³/h<Q<25 m³/h) were fit by $\Delta\varphi=R_{exp}I+cQB_{ctr}$ with R_{exp}=40.7 mΩ and c=31.62 m⁻¹; and by $\Delta p_{t61}=a_{61}IB_{ctr}-K_{61}Q^2$ with a_{61}=14.57 m⁻¹ and $K_{61}=3.54\times10^7$ Pa.m⁻⁶.s². To add the sixth half turn, we multiply the first term of Δp_{t61} by 1.195 (from our magnetic field calculation) and the second by 1.2 (=6/5). we also take into account that $B_{ctr}=B/1.17$ and $Q=UL_JL_B$. Our experimental laws become:

$$\begin{cases}\Delta\varphi=R_{\exp}I+d\,U\,B\\ \Delta p_{t\,pump}=a\,I\,B-\left(k_{loss}/2\right)\rho\,U^2\end{cases} \text{ with } \begin{cases}R_{\exp}=40.7\,m\Omega \text{ and } d=84.3\,mm\\ a=14.88\,m^{-1} \text{ and } k_{loss}=0.711\end{cases} \quad (9)$$

Notice that a and R_{exp} differ from the values $1/L_B$=25.6 m⁻¹ and $L_J/(\sigma L_U L_B)$=32.9 mΩ that would be obtained for a straight channel, because of the different distribution of the current density in the helical channel. However d is close to the value L_J=80 mm that we would have in a straight channel. The loss coefficient also differs from the regular loss coefficient k_{th}=1.3 calculated for a straight pump channel with the same dimensions, at a flowrate of 20 m³/h.

Switching to dimensionless forms and constructing the pump efficiency, we obtain:

$$\Delta p_{ta}=a\,L_B\,I_a-\frac{k_{loss}}{2}U_a^2\,,\;\; \eta_{pump}=\frac{U_a}{I_a}\frac{a\,L_B\,I_a-\left(k_{loss}/2\right)U_a^2}{\left(R_{\exp}\sigma\,L_U\,L_B/L_J\right)I_a+\left(d/L_J\right)U_a} \text{ for } \begin{cases}0.19<U_a<3.4\\ 2.2<I_a<53\end{cases} \quad (10)$$

Those laws are drawn in Fig 3 together with the theoretical laws for straight channels, showing a qualitative agreement and also that the efficiency could only be improved with a lower current and a lower velocity (at the expense of a lower head produced by the pump!).

On our model boat, a diverging nozzle was tested (Fig 5), but it creates strong losses as shown by the boat speed, reaching 0.3 m/s without the transparent plate on the rear part, but only 0.2 m/s with it. A CFD study showed strong recirculations in the divergent when equiped with the plate, and smooth mixing with the outer flow when this plate was removed. Therefore, we adopt a straight outlet (α_o=1) to describe our model without plate.

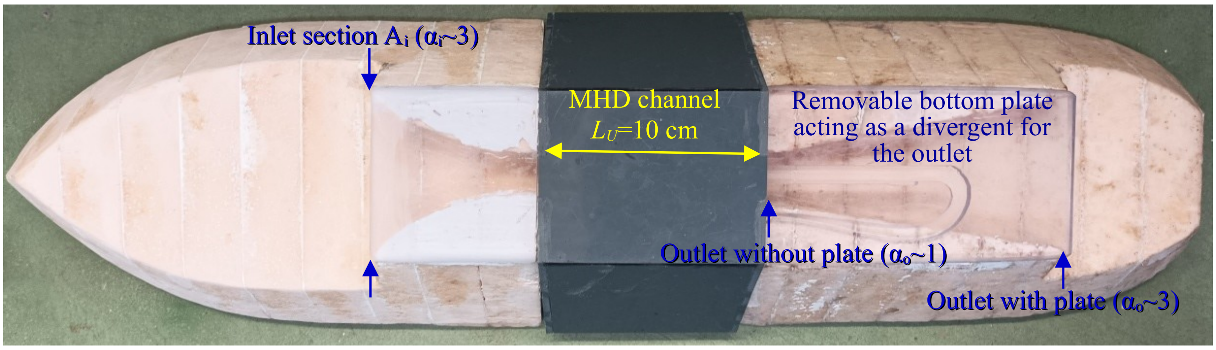

Figure 5. Model boat, bottom view

The cruise speed was experimentally 0.3 m/s for I=21 A. The loss coefficient of the channel (without diverging nozzle) was estimated to k_{tot}=1 (among which k_{loss}=0.8 for the mhd channel). Applying our model (Eq. 8) then gives the correct boat speed for C_D=0.57, what seems reasonable considering the sharp edges of the hull. The model also gives U/V_∞=2.81 (from Eq. 7), showing that the inlet section is almost adapted to the cruise speed ($\alpha_i \sim U/V_\infty$). Considering the pump part, we have U_a=5.1×10^3, I_a=1.5×10^7, Δp_{ta}=4.7×10^6 (see Eq. 4), and η_{pump}=0.011 %. Our model boat is far out of the interesting zone plotted in Fig 3.

Upscaling such a boat is favourable, but insufficient since U_a is only proportionnal to L^{-1}: with L_U=10m and everything else unchanged, we would have U_a=51, which is still too high to have a reasonable efficiency. Increasing B is also needed, i.e. we have to use superconducting magnets. For example with B=10 T we have U_a=1.9 for L_U=5 m and U=5 m/s. Then a pump efficiency of 27 % (for I_a=3.6) is reached if k_{loss}=0.8, but this loss coefficient decrease for large scale propellers (our model boat is penalized by a very strong relative rugosity of its channel). For B=10 T, L_U=5 m, k_{loss}=0.4 and U=5 m/s, we reach η_{pump}=41 % at I_a=2.3. With that superconducting pump, reduced drag and internal losses (C_D=0.25, k_{tot}=0.5), the boat scaled up by a factor 50 reach a cruise speed V_∞~3.2 m/s (Eq. 8).

5. Conclusion. A simple model is proposed here to describe MHD propulsion, where the MHD channel (condidered as a pump) is reduced to 3 dimensionless parameters, from which we can estimate the pump efficiency (i.e. the major part of the losses). Using this simple model may orient the pre-design phase of MHD thrusters with quick turnaround time.

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REFERENCES

[1] Y. SASAKAWA (ed). *Yamato-1, world 1st superconducting MHD propulsion ship* (Ship & ocean foundation, 1997).

[2] Y. DELANNOY, V. ROCHE. Elamhyd, électrochimie des actionneurs magnétohydrodynamiques en eau de mer (*Carnot Energies Day*, Grenoble, 2021)

[3] I. MONTERO ET AL. MagnetoHydroDynamics experiments with a helical channel in a 4T Superconducting Magnet. 13th PAMIR conference, Carry-le-Rouet, France (2024)