Contribution of coherent vortices such as Langmuir cells to wind-driven surface-layer mixing

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Abstract.
The wind blowing over the water surface causes, even for low windspeeds, Langmuir circulation in addition to shear-generated turbulence. Both these mechanisms mix the upper layer of oceans and lakes, but since the mixing efficiency of Langmuir cells is unknown, their relevance to mixed layer deepening is still an open question. In order to estimate the contribution to mixing by Langmuir vortex-cells relative to shear-induced mixing, we employ results on entrainment rate obtained from laboratory experiments with Taylor vortex cells. These are coherent horizontal vortices analogous to Langmuir cells. To relate the two, we define a surface friction velocity u_* that would be necessary to drive cells of strength equivalent to the Taylor vortices. It is then shown that up to a Richardson number of $Ri_* \approx 50$ layer deepening is predominantly caused by shear-generated turbulence, whereas for $Ri_* > 50$ the contribution by coherent Langmuir cells dominates the mixing process. For Richardson numbers $Ri_* > 120$, the entrainment rate decreases, but there is no criterion for the arrest of mixing by Langmuir cells as was previously assumed. The present results confirm observations that shear-generated turbulence dominates during initial layer deepening under relatively weak buoyancy effects, and that subsequently Langmuir-cell-mixing dominates the mixed layer deepening. The entrainment rates obtained from laboratory experiments are discussed and predict realistic values for the mixed-layer deepening.

1. Introduction

Mixed layer deepening has important consequences for general ocean circulation and climate models, as well as for the dynamics of lakes and reservoirs. Among the various surface-induced mixing processes such as shear, convection and precipitation, a mechanism that has drawn increasing interest is Langmuir circulation. Langmuir circulation (Langmuir, 1938) is caused by the combined action of wave-induced Stokes drift and wind-induced shear (Craik & Leibovich 1976; Garrett 1976; reviews of Leibovich 1983 and Thorpe 2004). The cells arise after several minutes of wind and wave forcing for wind speeds larger than 3 to 5 m/s and, depending on wind speed, may have downwelling velocities in a range of approximately 1 to 20 cm/s (see Table I). Their widespread occurrence for relatively weak winds suggests that these vortices may provide an important mechanism for the mixing of the upper layer of oceans, seas and large lakes (Weller et al. 1985, Wüest & Lorke, 2003). Observations (Li et al 1995; see Thorpe 2004 for a review) as well as large eddy simulations (Li et al. 2005) suggest that both, wind-induced shear and Langmuir circulation are often dominant processes for the mixed layer deepening, but their individual contribution to the mixing process is still unknown.

The entrainment rate across a density interface or a stratified layer varies with each dynamical process and is likely to be different for shear-generated turbulence and Langmuir turbulence. In order to discern the relevance of each mixing process, we consider the entrainment rates of these individual mixing processes. The entrainment rate due to shear generated turbulence has been fairly extensively investigated, but the entrainment rate by continuously driven horizontal vortices such as by Langmuir vortices is not unambiguously established. In a recent experimental study, we have studied the entrainment rate by Taylor vortices at a density interface in a Taylor-Couette flow. It is of interest to examine this entrainment rate in view of estimating entrainment by Langmuir vortices.

With respect to mixing across a density interface there is a loose, but physically plausible analogy between Taylor vortices and Langmuir vortices: (i) both are coherent turbulent vortices that are continuously driven and are aligned parallel to the density interface; (ii) a friction velocity u_* can be defined for Langmuir vortices which is related to the maximum downwelling velocity w_{dn}. This downwelling velocity is taken to be the same in Taylor and Langmuir vortices and relates the Taylor vortices to the friction velocity u_*; (iii) the characteristic length scale is the vortex size, equal to the mixed layer depth. The interest of the present analysis is the possibility of identifying quantitatively the mixing rate by coherent vortices such as Langmuir vortices with respect to mixing by shear-generated turbulence. Concerning mixing by Langmuir vortices, former studies assumed implicitly the vortices to be "laminar", i.e. there is only one vertical length-scale involved in the mixing; that is the characteristic scale at which the overturning motion of the vortex cell is arrested by stratification (Li et al. 1995; Li & Garrett, 1997). However, the results obtained with Taylor vortices indicate that mixing continues because the vortices are turbulent and contain a range of overturning scales. At large Richardson numbers, the overturning eddies become smaller and smaller so that the mixing continuous at a lesser rate. In previous studies on mixing by Langmuir vortices, no detailed comparison with mixing by shear turbulence has been given. The main object of the present paper is to give this comparison.

In Section 2, we will first discuss the criteria for mixing induced by Langmuir circulation and shear generated turbulence for a typical ratio between downwelling velocity and

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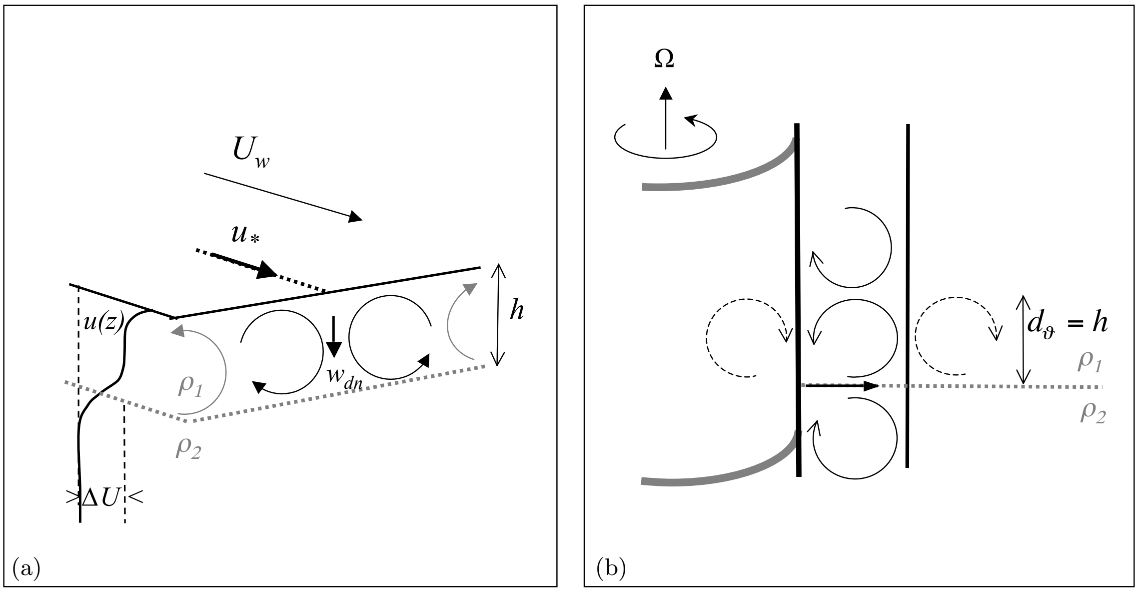

Figure 1. Schematic representation of Langmuir cells and the wind-induced shear flow (a), and of (b) Taylor-Couette flow with Taylor vortices eroding a density interface.

friction velocity suggested by Li et al. (1995). In Section 3, laboratory experiments on mixed-layer deepening by Taylor vortices are analyzed in the light of Langmuir vortices, and in Section 4 laboratory experiments on mixed-layer deepening by pure shear flows are considered. In Section 5 the results are discussed in the context of mixing in turbulent shear flows and Langmuir turbulence.

2. Mixing by Langmuir circulation and shear instability

Langmuir circulation strongly varies in space and time, making it difficult to estimate its global effect on mixing and to characterize its relevance with respect to other processes (see review of Thorpe 2004). It has been suggested that mixed-layer deepening by Langmuir circulation stops when (Li et al. 1995) the Froude number

$$Fr = \frac{w_{dn}}{(h\Delta b)^{1/2}} \leq C \qquad (1)$$

where C is of order 1. Here w_{dn} is the maximum downwelling velocity, $\Delta b = g\Delta\rho/\rho$ is the buoyancy jump at the base of the mixed layer and h is the mixed-layer depth. For C, Li et al. (1995) suggest 0.9 for a two-layer stratification and 0.6 for a linearly stratified fluid (Li & Garrett 1997). In order to differentiate the contributions of Langmuir circulation and shear-generated turbulence to the mixing process, the first step is to evaluate the relation between wind velocity U_w, the surface friction velocity u_* and the maximum downwelling velocity w_{dn}. For a typical wind speed of 10 m/s, the maximum downwelling velocity is of the order of 10 cm/s. The ratio w_{dn}/u_* between downwelling velocity and friction velocity is subject to an uncertainty. Typical values suggested by Li et al. (1995) that are used here are $u_* = 1.3 \cdot 10^{-3} U_w$ and $w_{dn} = 8.3 \cdot 10^{-3} U_w$ and thus $w_{dn} = 6.4 u_*$.

Downwelling is confined to narrow regions with maximum velocity at a certain depth and at a certain position between two cells of varying strength. In particular under storm conditions, the measurements of this value show large variations. Li et al. (2005) investigated the vertical velocity variance as a function of the Langmuir turbulence number, using large eddy simulations. The Langmuir turbulence number represents the ratio between friction velocity and the Stokes drift, U_s, due to wave action, $La_t = u_*/U_s$, implying weaker Langmuir cells relative to shear with increasing La_t (see McWilliams et al. 1997). The variance of the vertical velocity, w_{rms}^2/u_*^2 continuously decreases with the Langmuir number La_t, with a transition from Langmuir to shear turbulence at $La_t = 0.7$ (Li et al. 2005). Observations are only known for $La_t \leq 0.7$ for which Li et al. (2005) find the extremal values of w_{rms}, $0.8 < w_{rms}^2/u_*^2 < 3.25$ so that $4.5u_* < w_{dn} < 9.4u_*$. For $La_t \geq 0.7$ the ratio $w_{rms}^2/u_*^2 \approx 0.8$ and remains approximately constant with La_t.

Table 1 shows the maximum downwelling speed w_{dn} for different wind speeds. In order to estimate the variation of w_{dn} we use the root mean-square downwelling speeds which show generally much less scatter. From the values in Table I, we obtain $\bar{w}_{rms} = (1.6 \pm 0.3) \cdot 10^{-3} U_w$. Using the estimation for w_{dn} mentioned above, $w_{dn} = 5.2 \cdot \bar{w}_{rms}$, the observed limits for w_{dn} are $5.6u_* < w_{dn} < 8.7u_*$ (see the values for w_{rms}/u_* in Table I).

Though we use here observational data, results calculated with the larger range of w_{dn} found numerically by Li et al (2005) are represented in addition (see Figure 2) and indicate the largest range of uncertainty in the ratio w_{dn}/u_*.

2.1. Richardson numbers in use

For shear-driven mixing Price et al. (1986) determined two stability criteria. One is the usual stability of a stratified shear layer for which the gradient Richardson number criterion for stability is according to linear theory:

$$Ri_g = \frac{g\partial\rho/\partial z}{\rho(\partial U/\partial z)^2} \geq 0.25\,. \qquad (2)$$

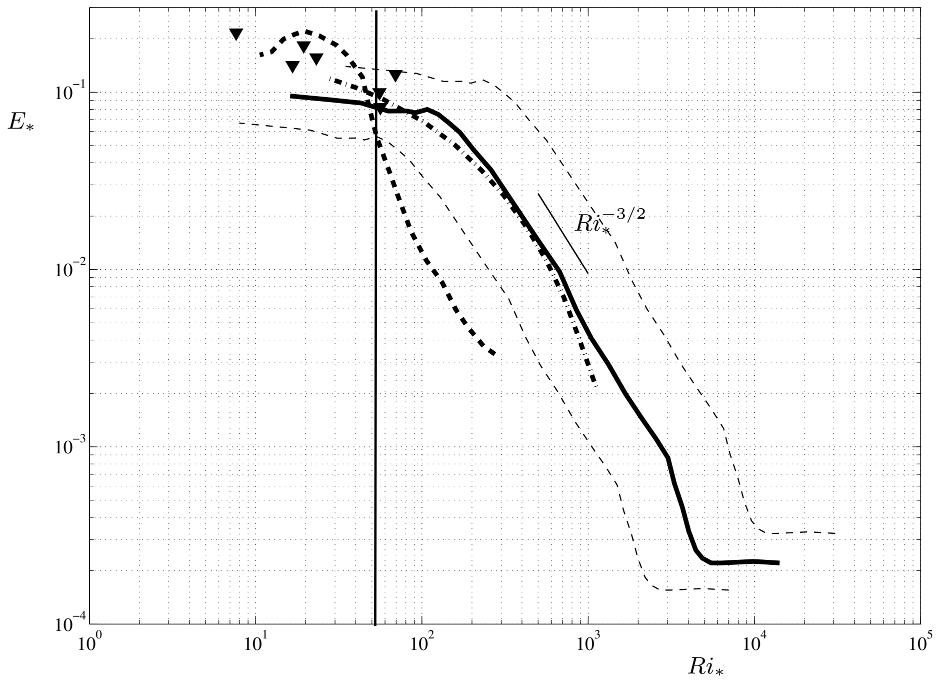

Figure 2. Comparison of dimensionless shear-generated turbulent mixing rate, $E_* = u/u_*$ with mixing by coherent vortices as a function of $Ri_* = \Delta bh/u_*^2$. The mixing rate by coherent vortices (turbulent Taylor vortices in analogy with Langmuir vortices) (thick solid line) is represented by the average of the experimental points of figure 2 of Guyez et al. (2007) with their rms velocity $U_\vartheta = w_{dn}/1.8$ and the relation $w_{dn} = 6.4u_*$ to obtain E_* and Ri_* ; shear- and vortex-generated mixing from Kantha et al. (1977) (dash-dotted line), and pure shear flows of Piat & Hopfinger (1981) (▼), and Strang & Fernando 2001 (thick dashed line). The limits (thin dashed lines) represent the extremal values $4.5u_* < w_{dn} < 9.4u_*$ found in large-eddy simulations by Li et al. (2005) for $La_t < 0.7$. The vertical line indicates the cross-over from mixing by shear to Langmuir circulation at $Ri_* \approx 50$.

This assumes that the density layer thickness is equal to or larger than the shear thickness. When the respective thicknesses are opposite or are dissymmetric, the critical gradient Richardson number differs from 0.25 (Strang and Fernando, 2001). Higher values of critical Richardson numbers are also predicted by nonlinear stability theories (see Abarbanel et al 1984). In the Price et al. model (1986) mixed layer stability is assured when

$$Ri_b = \frac{\Delta bh}{(\Delta U)^2} \geq 0.65 \quad (3)$$

where ΔU is the velocity difference across the density interface, i.e. the velocity equivalent of Δb. Li et al. (1995) used (1) and (3) to determine the possible importance of mixing by Langmuir circulation. This gives for Langmuir mixing the relation:

$$w_{dn}/C \geq \sqrt{0.65}\Delta U\,. \quad (4)$$

It is of interest to express the two Richardson-number criteria for interfacial shear-instability (2) and for mixed-layer stability (3) in terms of the surface-friction velocity, that is in terms of the Richardson number $Ri_* = \Delta bh/u_*^2$. For doing this it is necessary to find a relation between the velocity across the mixed layer, ΔU, and u_*.

From relation (4) and $w_{dn} = 6.4u_*$ we obtain for a two-layer stratification with $C = 0.9$, $u_* \approx 0.11\Delta U$ and for a linear stratification $u_* \approx 0.075\Delta U$ with $C = 0.6$. With these values for $u_*/\Delta U$, we get from (3) for mixed layer stability and, according to (1), arrest of mixing by Langmuir cells in a two-layer and linear stratification, respectively,

$$Ri_{*b} = \frac{\Delta bh}{u_*^2} \geq 50 \text{ when } C = 0.9 \quad (5)$$

$$Ri_{*b} = \frac{\Delta bh}{u_*^2} \geq 114 \text{ when } C = 0.6\,. \quad (6)$$

As will be shown in Section 3, there is in fact no critical Froude or Richardson number but mixing by Langmuir circulation continuous above these critical values.

An independent estimation of $u_*/\Delta U$ can be obtained from turbulent plane Couette flow since the constraints by the density interface have an effect similar to a wall when the stratification is strong. In both cases the mean velocity-profile tends towards an S-shaped profile with ΔU being about half the surface velocity (see Figure 1a). In a plane Couette-flow over a smooth surface, $u_* \approx 0.05U_c$ where U_c is the centre velocity (Robertson & Johnson, 1970), which here is the velocity in the centre of the mixed layer. For strong stratification we then have $\Delta U \approx U_c$ and since the surface boundary can be considered fully rough $u_* \approx 0.08\Delta U$ by analogy with a fully rough versus a smooth boundary layer. This value of 0.08 is well within the values estimated above.

The determination of interfacial stability in terms of u_* is less straight forward, because we have to consider the entire Richardson number range. When substituting in (2) $\partial U/\partial z = \Delta U/\delta$, $\partial\rho/\partial z = \Delta\rho/\delta$ and $\delta = c_1 h$, where δ is the interface thickness, we obtain for interfacial stability $Ri_g = \frac{\Delta bc_1 h}{(\Delta U)^2} \geq 0.25$. Here, ΔU is again the velocity difference across the interface, which is in general

$\Delta U = c_2 u_*$ where c_2 decreases with increasing Richardson number. Substituting this expression for ΔU we get

$$Ri_g = \frac{c_1}{(c_2)^2} Ri_* \geq 0.25. \quad (7)$$

If we take $c_2 = 1/0.075$ and, consistent with (5), $Ri_* = 114$, then shear instability, and according to (3) mixed layer stability, is assured when $c_1 \geq 0.39$. When $c_2 = 1/0.11$ and $Ri_* = 50$, stability requires $c_1 \geq 0.41$. These relatively large values are not unrealistic at these low values of Ri_*.

3. Mixing by Taylor vortices and relation with Langmuir circulation

Some new information about mixing by Langmuir circulation can be obtained by analyzing recent experimental results by Guyez et al. (2007) (GFH) on mixing across a density interface by turbulent Taylor vortices. Although Taylor vortices are driven by centrifugal instability whereas Langmuir circulation is driven by surface wave conditions, both are coherent, turbulent vortices with their axes parallel to the practically horizontal pycnoclines. At present, the study by GFH is the only one that considers mixing at an interface by such horizontal, continuously driven vortices.

Langmuir circulation consists of an array of horizontal vortex pairs (Figure 1a) whereas the Taylor-Couette flow consists of an array of vertically arranged vortex pairs (Figure 1b). However, only the vortex adjacent to the density interface is active in mixing across the interface so that the size of the vortex $d_\vartheta = h$ represents the main mixing length; the other vortices just distribute the density in the layer above this vortex. Furthermore, the fact that there are vortices on both sides of the interface is of no importance for the mixing. There is no correlation between the two sides as was shown by Turner (1968) for turbulent mixing; it only serves to keep the interface at a fixed position. As is indicated in Figure 1b, the walls in the Taylor-Couette device can be considered as symmetry planes (the boundary layers on the walls being negligible) with image vortices on the other side of the walls. There is a maximum downwelling velocity in Taylor vortices u_{max}, which is taken equal to w_{dn} at the inner wall (Dong, 2007). The mixing process can thus be considered analogous to that of a Langmuir vortex flow with shear-generated turbulence being absent. The Taylor vortex flow is spiraling as in Langmuir circulation due to the mean azimuthal (horizontal) velocity. It needs to be mentioned that in the experiments of GFH the radius of the inner cylinder was sufficiently large (20 cm with a gap width of 5 cm) to neglect possible curvature effects on mixing.

To relate the mixing by Langmuir and by Taylor vortices to the surface friction velocity u_* we assume that the friction velocity that would be required to maintain Taylor vortices (of strength $u_{max} = w_{dn}$) is the same as for Langmuir vortices, i.e. $w_{dn} = 6.4u_*$. In Figure 2 the mean of the experimental value of the entrainment by the Taylor vortices is shown in terms of an equivalent friction-velocity i.e. $E_* = u_e/u_*$ as a function of $Ri_* = \Delta bh/u_*^2$. Note that when represented in this way the entrainment curve is only shifted, as expected, and the flow remains of course shear free. The friction velocity relates the maximum downwelling velocity in the Taylor vortices to a specific wind speed.

4. Comparison with shear generated turbulence

4.1. Annular shear flow

In experiments with shear flows in annular tanks, the upper layer was driven by a rotating lid at the surface that generated a shear flow across the interface (see Kantha et al. 1977; Scranton & Lindberg 1982, and Deardorff & Yoon, 1984). In these flows, in addition to the shear-generated turbulence, the centrifugal force drives a secondary circulation that extends over the depth of the mixed layer (of depth h) and is comparable to a Taylor vortex or Langmuir circulation. The entrainment coefficient and Richardson number are respectively defined as $E_w = (dh/dt)/u_*$ and $Ri_* = \Delta bh/u_*^2$, with the mixing length being taken equal to the depth of the fluid h, corresponding to the size of the secondary eddies driven by the centrifugal force, or the eddies driven by the shear at the surface. In Figure 2 the entrainment data of Kantha et al. (1977) are compared with the entrainment rate by Taylor vortices, scaled with the corresponding u_*. It is seen that when $Ri_* < 50$ the entrainment rate is higher than in the Taylor-vortex flow. This is to be expected because of shear-generated turbulent mixing and interfacial instability in addition to the secondary circulation. However, for $Ri \geq 100$ mixing by coherent vortices dominates, in good agreement with the entrainment rate by Taylor vortices (see Figure 2). The shear does not increase the entrainment rate by coherent vortices.

4.2. Shear flows without secondary circulation

Piat and Hopfinger (1981) considered a turbulent boundary-layer topped by a density interface in a two-layer stably stratified flow. The two-layer density stratification was established by heating the upper layer. The turbulent boundary layer was generated at a rough bottom and its growth in thickness, $h(x)$, was affected by the two-layer stratification. The mixing length is set by the eddies in the turbulent boundary layer and is limited by its maximum size $h(x)$. The entrainment coefficient and Richardson number varied with distance and were, respectively, defined as $E_* = u_e/u_*$ where $u_e = U_0\ dh/dx$ with U_0 the upper-layer velocity, and $Ri_* = \Delta bh/u_*^2$. The data in Figure 2 shows further increasing entrainment rates with decreasing Richardson number, and extends the values of Kantha et al. (1977). This indicates clearly that at low Richardson number ($Ri_* < 50$) shear-induced mixing dominates.

Another example concerns a mixing layer at an interface between two different density fluids, of which the upper layer moved with an approximately uniform velocity (Strang & Fernando 2001). This flow was generated in the Odell-Kovasznay apparatus. The density interface thickness and the shear thickness are unequal and not centered so that Kelvin-Helmholtz and Hölmböe instabilities occur at low *Ri*-numbers. These instabilities enhance entrainment into the upper layer. The location of the interface centre was used to determine the entrainment velocity. Since the flow is not driven by surface friction, the shear in between the two layers can not directly be compared to the shear flow induced by surface friction. As a first approximation, we can take the velocity u_{rms} in the mixed layer equal to the surface-friction velocity u_*. Furthermore, the integral length-scale of the turnover motions at the interface, L_{11}, represents the scale of the energy containing eddies that contribute to the mixing process (see Strang & Fernando 2001, p9). This is the most appropriate scale for the mixing length in these experiments so that L_{11} is the equivalent of the scale h in the other experiments. Thus one obtains a Richardson number $Ri_* = \Delta bL_{11}/u_*^2$ and entrainment coefficient $E_* = u_e/u_*$, noting that $u_* = u_{rms}$. The results, shown in Figure 2, are in agreement with the results of Piat & Hopfinger (1981) when $Ri_* < 50$, thus confirming the coherence of the scaling.

It is seen in Figure 2 that for low Ri-numbers both, shear and Langmuir circulation, contribute to the mixing process with shear-produced turbulent mixing being dominant. According to the Taylor-vortex results, the entrainment rate by Langmuir circulation is about half the value of shear-generated mixing. With increasing Ri_*, the entrainment

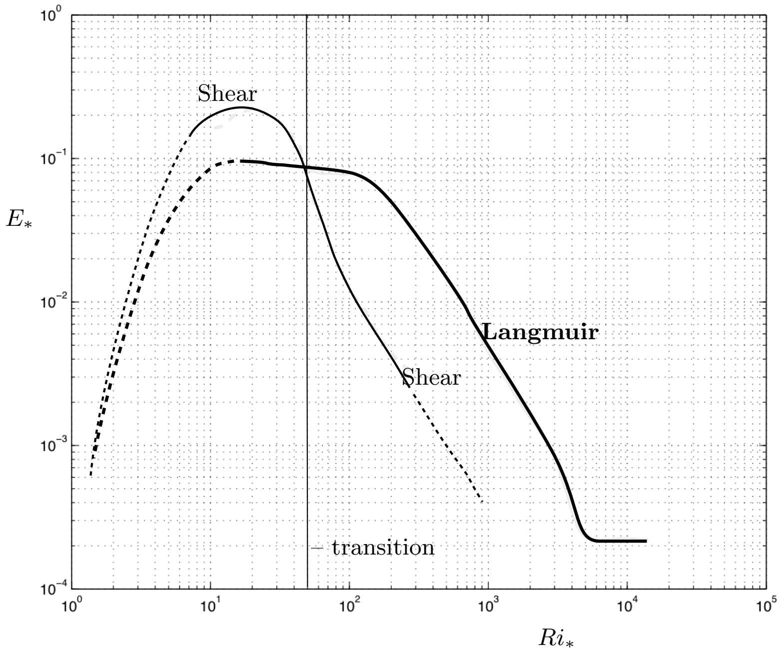

Figure 3. Summary of entrainment rates by shear turbulence (thin line) and by turbulent Langmuir cells (thick line) in accordance with figure 2. The transition is at $Ri_* \approx 50$. For comparison, Fr=1 with the Froude number in (1) corresponds to $Ri_* = 37$. The dashed lines indicate the generally expected continuation of these lines.

rate decreases and interfacial Kelvin-Helmholtz instability is arrested by buoyancy. The cross-over Richardson number between shear-dominated and Langmuir-cell dominated mixing is, according to Figure 2, about $Ri_* \approx 50$. The present results show that for $Ri_* > 50$ mixing by Langmuir cells does not stop, but on the contrary dominates the mixing process.

The entrainment coefficient for coherent vortices remains nearly constant up to $Ri_* \approx 120$ which corresponds to $C = 0.6$ in (1). This implies that overturning at the scale h is possible for Froude numbers $Fr \leq 0.6$ as in a linearly stratified fluid. The experiments by GFH indicate that for these relatively low values of Ri_* the interface is not sharp and is of the order of the vortex size.

5. Discussion and conclusions

From the results shown in Figure 2 two conclusions can be drawn concerning mixing by Langmuir circulation. One is that entrainment by coherent vortices is by engulfment and nearly unaffected by stratification as long as $Ri_* \leq 120$ (corresponding to a value of $1/C^2 \approx 3$, hence $C \approx 0.6$ in (1)). However, when $Ri_* \geq 120$, mixed layer deepening by Langmuir circulation does not stop, but continues for large Richardson numbers at a slower rate with $E_* \approx 0.15 Ri^n$ with $n \leq -3/2$. There is no critical Froude number for the arrest of entrainment by coherent (Langmuir) vortices as is implied by (1). The reason is that Langmuir cells are turbulent and contain energetic eddies at scales less than h; when overturning at the scale h is prevented by buoyancy (as is implied by (1)), vertical mixing is still possible. The second result is the cross-over at $Ri_* \approx 50$ of dominant mixing by coherent vortices. Below this value mixing by shear is dominant and above, Langmuir circulation appears to dominate mixed-layer deepening contrary to the suggestion by Li et al. (1995) and Li & Garrett (1997). This behavior is summarized in Figure 3 which is consistent with the results shown in Figure 2.

Table 1. Observations on layer deepening rate, maximum and root mean square downwelling velocity, and maximum wind-speed or range of wind speeds U_w (in m/s). According to the estimation of Li et al. (1995) $u_* = 1.3 \cdot 10^{-3} U_w$ m/s.

Reference	Deepening (m/h)	w_{dn} 10^{-2} (m/s)	w_{rms} 10^{-2} (m/s)	U_w (m/s)
Weller & Price 1988		$(0.2U_w + 1) < w_{dn} < (1.7U_w + 5)^{(1)}$		3 to 15
Thorpe et al. 1994			$0.17U_w + 0.14$ $^{(3)}$	4 to 22
Smith 1998	3.1	$7^{(2)}$	$0.12U_w$ $^{(3)}$	15
D'Asaro & Dairiki 1997	3.0	12	2	12
			$1.2u^* < w_{rms} < 1.7u^*$	
D'Asaro 2001	3.2		4	12 to 13
			$1.16u^*$	
Tseng & D'Asaro 2004			$1.07u^*$	12 to 13

[1] Range, estimated from the overview graph of figure 23 in Weller & Price 1988, and average $w_{dn} = 8.3 \cdot 10^{-3} U_w$.
[2] Approximated value from the data.
[3] Supposed that the mean surface convergence speed $\approx \frac{1}{2} w_{rms}$.

Observations generally report deepening rates of approximately 3 m/hr for wind speeds of approximately 10 m/s. With the values of Li et al. (1995) for u_* given in section 2, we obtain with $E_{*max} = u_e/u_* = 10^{-1}$ a maximum deepening rate of 4.7 m/hr. Langmuir circulation starts to be dominant for $Ri_* > 50$ which, for a pycnocline with typically $\Delta b = 3 \times 10^{-3} ms^{-2}$, corresponds to a depth of about 3 meter. When increasing to a depth of 25m, Ri_* increases to a value of approximately 440, so that the mean entrainment coefficient for this Ri-number interval is $E_* \approx 0.05$, corresponding to a deepening rate of $0.234U_w$ m/hr. For a wind speed of 13 m/s the upper layer thus deepens with a rate of 3.04 m/hr. This result is in very good agreement with the observations (see Table 1).

References

Abarbanel, H. D. I., Holm, D. D. Marsden, J. E. and T. Ratiu, 1984 Richardson number criterion for the nonlinear stability of three-dimensional stratified flow, *Phys. Rev. Lett. 52*, 2352-2355.

Craik, A. D. D., and S. Leibovich, 1976 A rational model for Langmuir circulations, *J. Fluid Mech. 73*, 401-426.

D'Asaro, E.A. and G.T. Dairiki, 1997 A rational model for Langmuir circulations, *J. Fluid Mech. 73*, 401-426.

D'Asaro, E.A. 2001 Turbulent vertical kinetic energy in the ocean mixed layer *J. Phys. Oceanogr 31*, 3530-3537.

Deardorff, J. W. and S.-C. Yoon, 1984 On the use of an annulus to study mixed-layer entrainment. *J. Fluid Mech. 142*, 97-120.

Dong, S. 2007 Direct numerical simulation of turbulent Taylor-Couette flow. *J. Fluid Mech. 587*, 373-393.

Garrett, C., 1976 Generation of Langmuir circulations by surface waves? A feedback mechanism. *J. Mar. Res. 34*, 117-130.

Guyez, E.J., J.B. Flór, and E. J. Hopfinger 2007 Turbulent mixing at a stable density interface: the variation of the buoyancy flux-gradient relation. *J. Fluid Mech. 577*, 127-136.

Kantha, L. H., Phillips, O. M. and R. S. Azad, 1977 On turbulent entrainment at a stable density interface. *J. Fluid Mech. 79*, 753 -768.

Langmuir, I., 1938 Surface motion of water induced by wind. *Science 87*, 119-123.

Leibovich, S., 1983 The form and dynamics of Langmuir circulations. *Ann. Rev. Fluid Mech. 15*, 391-427.

Li, M., K. Zahariev and C. Garrett, 1995. The role of Langmuir circulation in the deepening of the surface mixed layer. *Science 270*, 1955-1957.

Li, M. and C. Garrett, 1997. Mixed layer deepening due to Langmuir circulation. *J. Phys. Oceanogr. 27*, 121-132.

Li, M., C. Garrett and E. Skyllingstad, 2005. A regime diagram for classifying turbulent large eddies in the upper ocean. *Deep-Sea Research I 52*, 259-278.

McWilliams J. C., Sullivan P. P. and Chin Noh Moeng, 1997 Langmuir turbulence in the ocean. *J. Fluid Mech. 334*, 1-30.

Price, J. F., R. A. Weller, and R. Pinkel, 1986 Diurnal cycling: Observations and models of the upper ocean response to diurnal heating, cooling, and wind mixing. *J. Geophys. Res. 91*, 8411- 8427

Piat, J.-F. and E.J. Hopfinger, 1981 A boundary layer topped by a density interface. *J. Fluid Mech. 113*, 411 - 432

Robertson, J.M. and H.F. Johnson, 1970 Turbulence structure in plane Couette flow. *ASCE J. Engng Mech. Div. 96*, 1171-1182

Scranton, D. R., and W. R. Lindberg, 1983. An experimental study of cntraining stress-driven, stratified flow in an annulus. *Phys. Fluids 26* 1198-1205

Smith J.A., 1998 Evolution of Langmuir circulation during a storm. *J. Geophys. Res. 103* 12.649 - 68

Strang, E. J. and H. J. S. Fernando, 2001 Entrainment and mixing in a stratified shear flows. *J. Fluid Mech. 428*, 349 - 386.

Thorpe SA, Cure MS, Graham A and A.J. Hall, 1994. Sonar observations of Langmuir circulation and estimation of dispersion of floating bodies. *J. Atmos. Ocean. Technol. 11*, 1273 - 94

Thorpe, S.A., 2004 Langmuir circulation. *Ann. Rev.Fluid Mech. 36*, 55-79

Tseng, R.S. and E. A. D'Asaro 2004, Measurements of turbulent vertical kinetic energy in the ocean mixed layer from Lagrangian floats. *J. Phys. Oceanogr. 34*, 1984-1990.

Turner, J. S. 1968 The influence of molecular diffusivity on turbulent entrainment across a density interface. *J. Fluid Mech. 33*, 639-656.

Weller R.A., Dean J.P., Marra J, Price J.F., Francis E.A., and D.C. Broadman, 1985 Three dimensional flow in the upper ocean. *Science 227*, 1552 - 56

Weller R.A. and J.F. Price, 1988 Langmuir circulation within the oceanic mixed layer. *Deep Sea Res 35*, 711-47

Wüest A. and A. Lorke, 2003 Small-scale hydrodynamics in lakes. *Ann. Rev. of Fluid Mech. 35*, 373-412.

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