Estimation of Friction Parameters and Laws in Gravity currents by Data Assimilation in a Model Hierarchy

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Etudes et paramétrisation des flux turbulents pour les courants gravitaires océaniques par assimilation de données

Abstract: Results on the dynamics of a gravity current (Wirth, *Ocean Dyn.* **59**, 551, 2009) and on the estimation of friction laws by data assimilation (Wirth & Verron *Ocean Dyn.* **58**, 247, 2008) are combined to estimate the friction laws and parameters in non-hydrostatic numerical simulations of gravity current dynamics, using data assimilation and a reduced gravity shallow water model. We demonstrate, that friction parameters and laws in gravity currents can be estimated using data assimilation. The results clearly show that friction follows a linear Rayleigh law for small Reynolds numbers and the estimated value agrees well with the analytical value obtained for non-accelerating Ekman layers. A significant and sudden departure towards a quadratic drag law at Reynolds number of around 1000 is shown, in agreement with laboratory experiments. The drag coefficient obtained compare well to friction values over smooth surfaces.

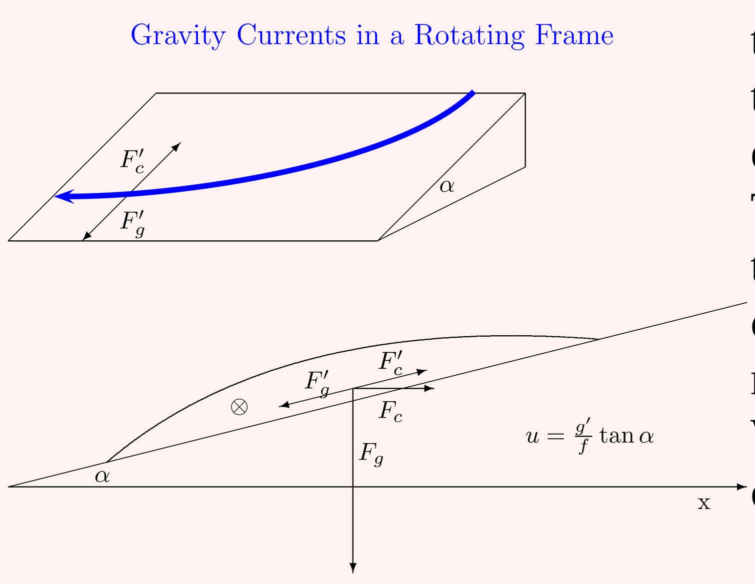
Gravity Currents in the Ocean

Buoyancy forces caused by density differences of fluids are the major source of fluid motion in nature. When these forces act adjacent to topography gravity currents are created. In the ocean the most conspicuous of such currents appear at sills and straits, connecting ocean basins or marginal seas to the ocean, in form of overflows. Important locations of such overflows are the Denmark Strait, the Faroes's Bank Channel, the Strait of Gibraltar, the Bosporus and Bab el Mandeb. The major part of the deep and intermediate waters of the world ocean and marginal seas have done at least part of their voyage to the deep in form of a gravity current. On its descent into the deep a gravity current is subject to bottom friction and mixes with surrounding waters, entrains surrounding waters pulling them into the deep, and in some occasions water from the gravity current is detrained and mixes with the surrounding waters. The turbulent friction, mixing, entrainment and detrainment not only determine the water mass properties of the gravity current, and the resulting intermediate and deep waters, but has a strong influence on its pathway. This formation process of the deep and intermediate waters is an important part of the thermo-haline overturning circulation which is key in determining our climate and climate variability. The aim of the here presented research is to further the fundamental understanding of oceanic gravity currents by determining the turbulent fluxes governing its behaviour and to obtain an efficient parametrisation for these fluxes in OGCMs by using data assimilation.

Dynamics of Oceanic Gravity Currents

Leading Order (geostrophy)

For the majority of oceanic gravity currents rotation plays an important role, as the time of descent exceeds largely one day. When neglecting dissipative processes, a gravity current approaches a geostrophic equilibrium travelling along isobaths with the constant speed for which the down slope force of gravity is perfectly balanced by the Coriolis force $u_G = g'/f \tan\alpha$

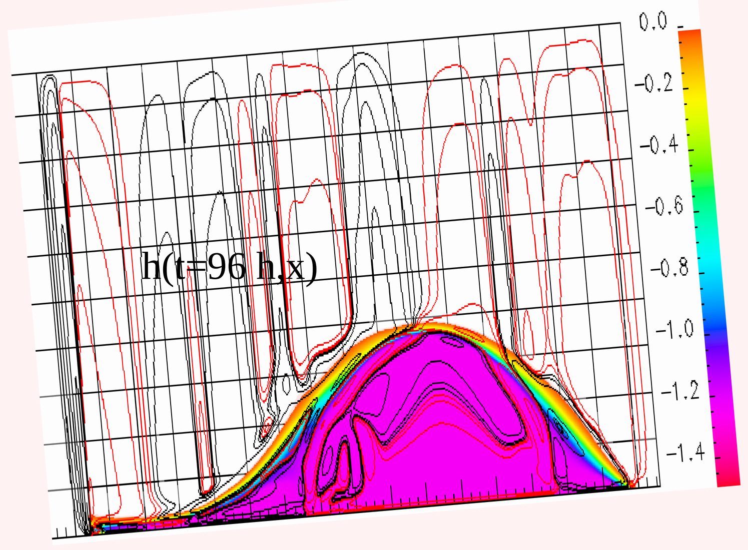


What makes gravity currents flow down-slope?

Turbulent fluxes of momentum and density perturb the geostrophic equilibrium and drain energy from the gravity current which is provided by potential energy as the gravity current descends the slope. These small scale processes determine the evolution of the gravity current. These turbulent fluxes can NOT be explicitly resolved in today's (and tomorrows) models of the global ocean circulation. We have to find efficient parametrisations and the correct parameter values for these turbulent fluxes

Friction does it!

Non-hydrostatic simulations



The data is taken from non-hydrostatic simulations (HAROMOD).

Parameter Estimation with:

Shallow Water model

The data is assimilated by a shallow water model in which the bottom friction is parameterised

$$\begin{aligned}
\partial_t u \; &+ \beta u \partial_x u - f v + g'(\partial_x h + \tan\alpha) &= -4\beta D u + \nu \partial_x^2 u, \\
\partial_t v \; &+ f u &= -D v + \nu \partial_x^2 v, \\
\partial_t h \; &+ u \partial_x h + h \partial_x u &= \nu \partial_x^2 h
\end{aligned}$$

$$D = (\tau + r/h^2 + c_D\sqrt{4\beta u^2 + v^2})/h.$$

Two friction laws: linear Rayleigh friction (τ and r), quadratic drag law (c_D) are implemented. The Coriolis-Boussinesq parameter β takes into account the vertical shear due to bottom friction.

&

Ensemble Kalman Filter

The first step consists in including the parameters to be estimated, in the augmented state vector:

$$\mathbf{x}(x,t) = (h(x,t), u(x,t), v(x,t), \tau, r, c_D)$$

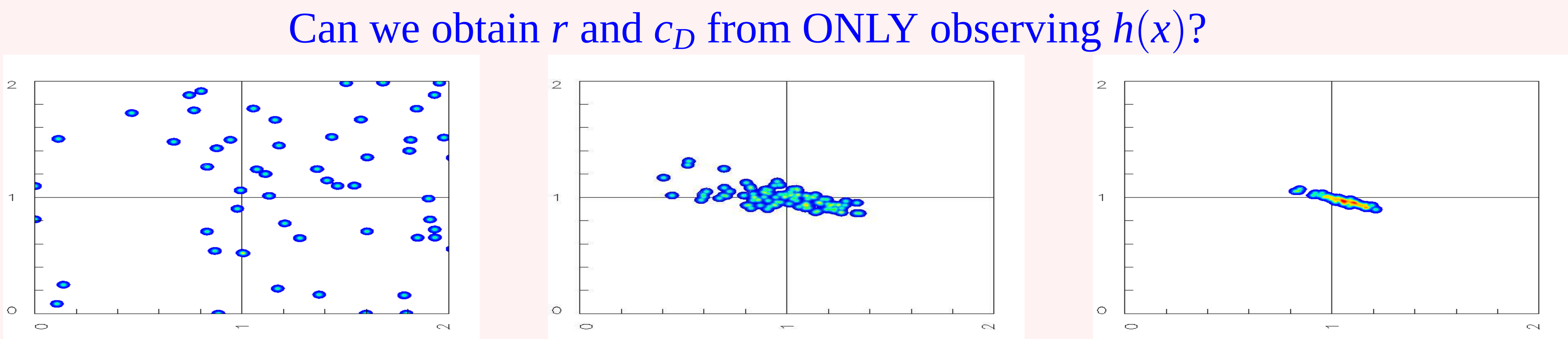
The only variable observed is the thickness of the gravity current $h(x,t)$. The EnKF (Ensemble Kalman Filter), which updates the ensemble mean and variance by updating each ensemble member, can then be applied on the augmented state vector:

$$\mathbf{x}_i^a = \mathbf{x}_\mathbf{i}^\mathbf{f} + \mathbf{K}\left(h^{obs} + \varepsilon_i - \mathbf{H}\mathbf{x}^\mathbf{f}{}_i\right)$$

H the observation matrix and **K** is the gain matrix, based on correlations between the components of the augmented state vector.

Feasibility

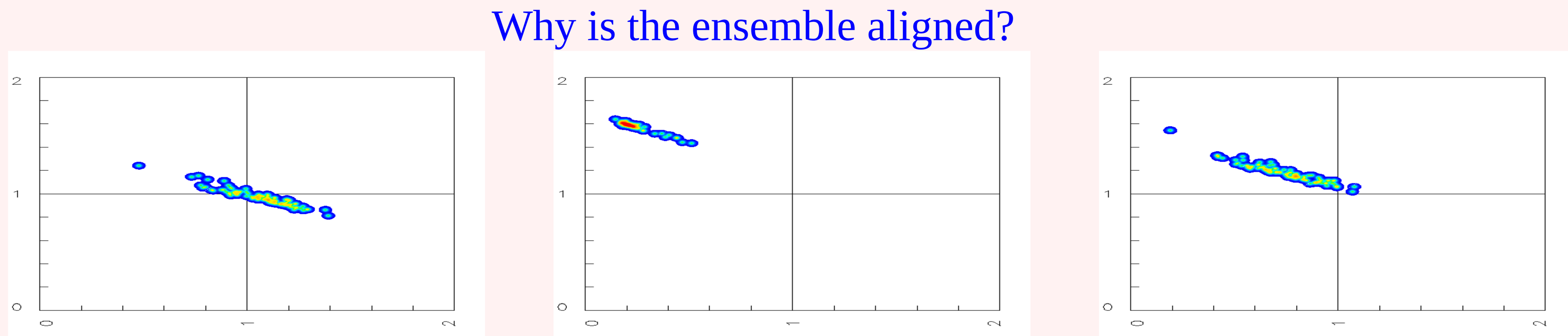
Can we obtain r and c_D from ONLY observing $h(x)$?



t=0 | t=7 days | t=7×7 days

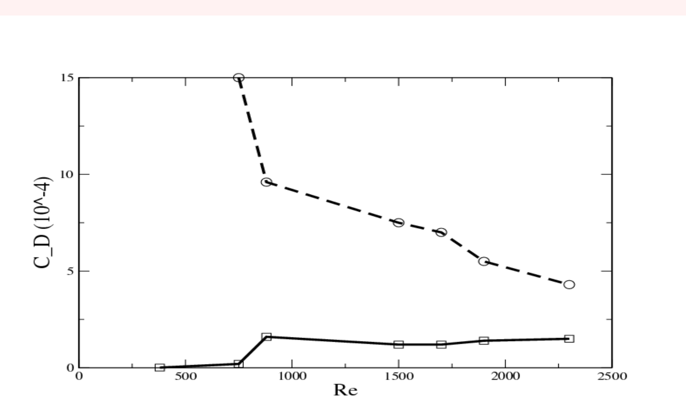
The ensemble in the $\tau \times c_D$ space during the identical twin data assimilation experiment. ($\tau_0 = 5.10^{-4}$, $c_D = 5.10^{-3}$, observation error $\sigma = 20$m data assimilation performed every hour). The convergence of the entire ensemble to the correct value demonstrates the feasibility of the estimation of friction parameters.

Why is the ensemble aligned?



Because it is easy to estimate the average friction $\langle D\rangle_{x,t}$, where: $D = (r + c_D|\mathbf{u}|)$, but it is more difficult to separate it into linear Rayleigh friction vs. quadratic drag law. We have a fast projection on the "slow manifold" corresponding to $\langle D\rangle_{x,t} = const$. followed by a slow convergence within the "slow manifold."

Results



The drag coef. c_D (full line) and the effective drag coef. $\tilde{c_D} = c_D + \tau/\sqrt{v_g}$ (dashed line), where v_g is the average geostrophic velocity, are presented as a funct. of the (Ekman layer) Reynolds number Re_{Ek}.

The results clearly show that friction follows a linear Rayleigh law for small Reynolds numbers and the estimated friction coefficient agrees well with the analytical value obtained for non-accelerating Ekman layers. A significant and sudden departure towards a quadratic drag law at Reynolds number of around 1000 is shown, in agreement with classical laboratory experiments (Nikuradse 1933, Schlichting & Gertsen 2000). Although the quadratic drag law is dominated by the linear friction at the Reynolds numbers considered in this study, the assimilation procedure clearly manages to detect its appearance and consistently estimate its value.

Conclusions

We demonstrate, that friction parameters and laws in gravity currents can be estimated using data assimilation. The drag coefficient obtained compare well to friction values over smooth surfaces. We also show that data assimilation is a powerful tool in systematically connection models within a model hierarchy. The here presented method can also be used to connect models to experimental data and can be applied to parametrisations of other processes in the ocean and elsewhere in science.

Perspectives

Turbulent fluxes of mass, that is: entrainment, detrainment and mixing, at the interface between the gravity current and the surrounding fluid, play an important role in the evolution of turbulent gravity currents. The determination of the parametrisations and the parameter values of these fluxes, using the here presented formalism are currently performed.