Inertia-Gravity waves generated by near Balanced Flow in 2 Layer Shallow Water Turbulence on the β-Plane

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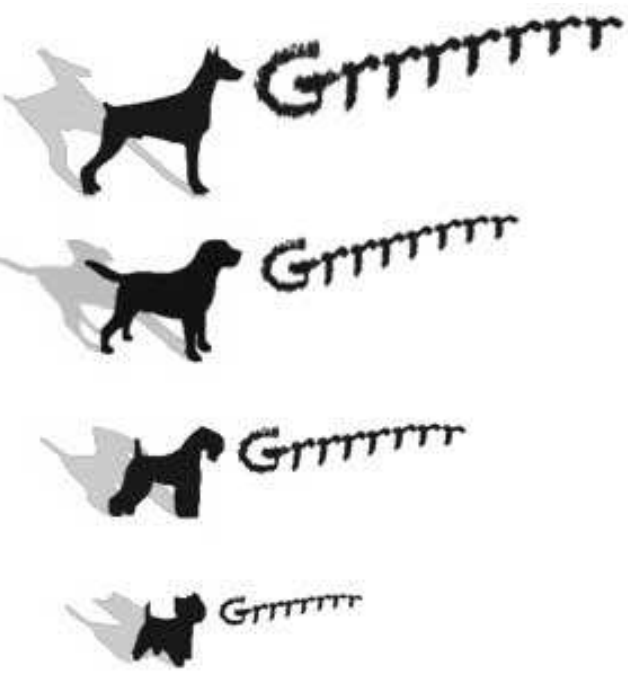
MEIGE / LEGI / CNRS

EGU- Wien, April 10, 2013

The Growl of near Balanced Flow
in 2 Layer Shallow Water Turbulence on the β-Plane



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Loss of Balance

(Hard) Loss of Balance by instability (Ford et al. 2000, Molemaker & McWilliams 2006)

- unstable stratification ($N^2 < 0$)
- sign change in absolut vorticity
- sign change in the difference of absolut vorticity and horizontal strain rate
- $Fr > 1$

bark

Spontaneous (Soft) Loss of Balance

- fuzzy manifold with thickness depending on Ro and Fr.

crowl

Analogy

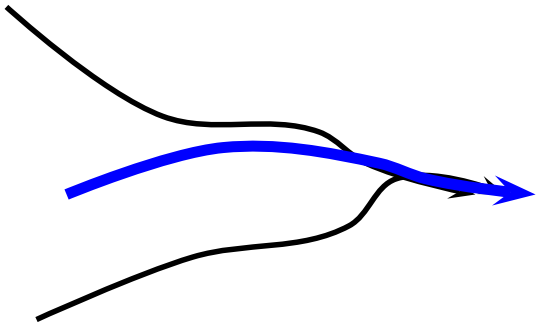
	Loss of Balance	Sound Emission
ref.	Ford et al. 2000	Lighthill 1952
hard l. of b.	Froude ≥ 1 Rossby ≥ 1	Mach number ≥ 1
Soft l. of b.	? ? ? ? ? ? ?	due to vorticity interaction with structures

Difference

energy cascade	inverse	direct
enegy dissip.	important	not important

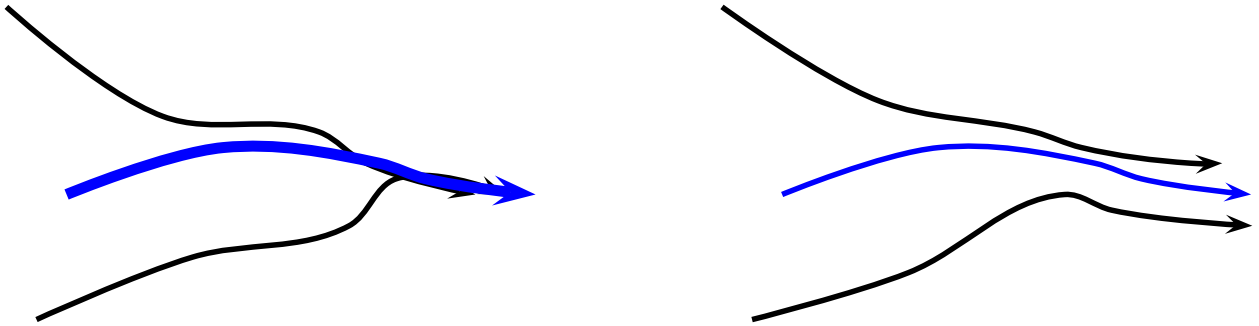
Phase Space

trajectory slow manifold



Phase Space

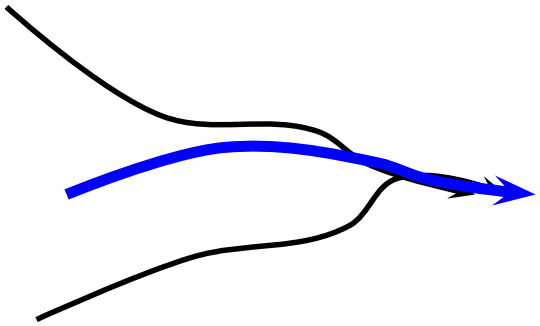
trajectory slow manifold

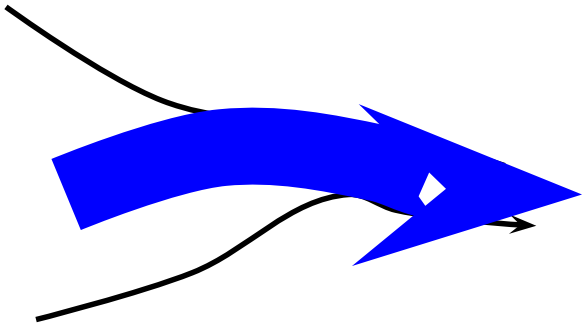


Phase Space, Fuzzy Manifold

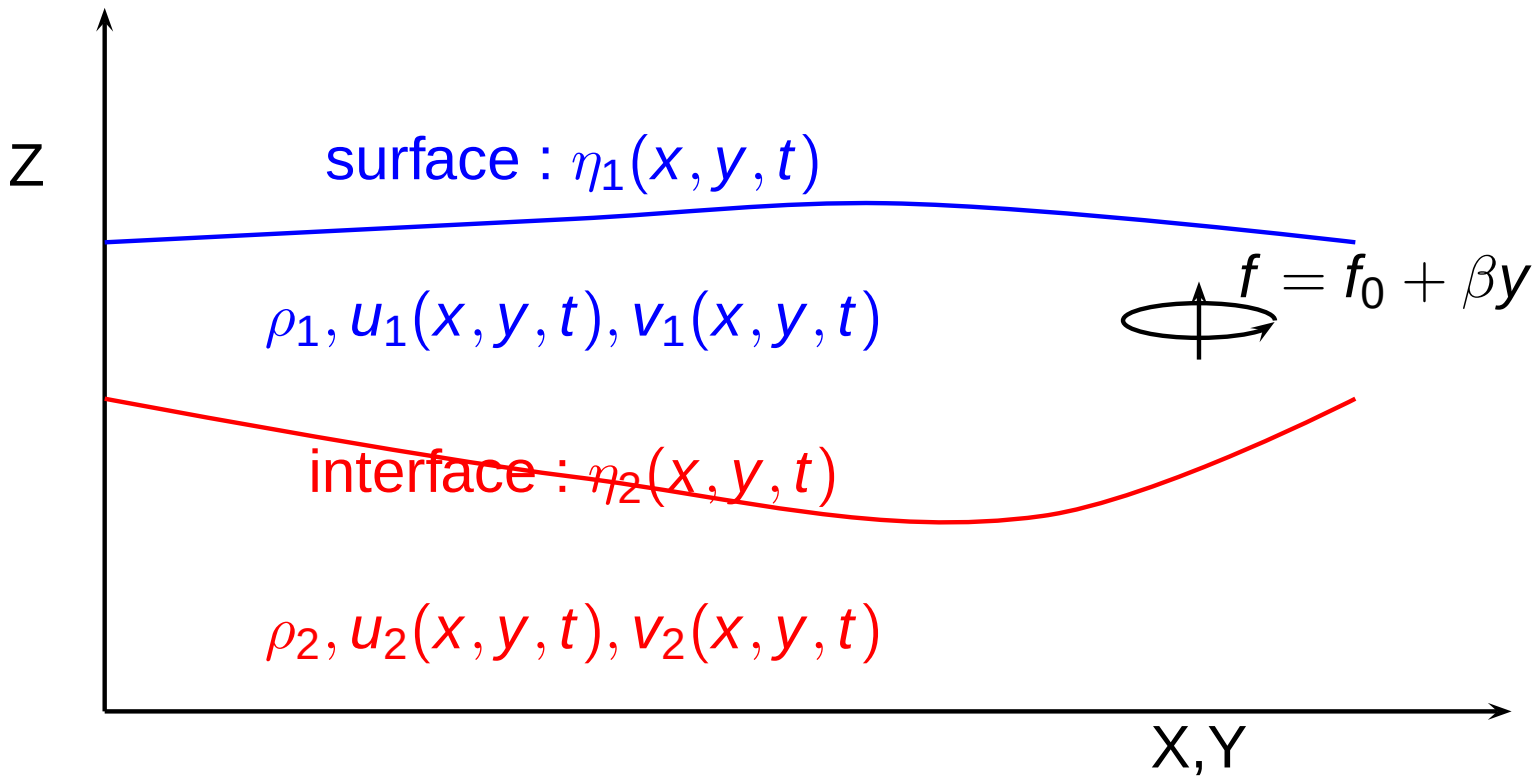
trajectory

slow manifold





Physical model



Physical Parameters

h_1	h_2	g	g'	f	β	L_x	L_y
600m	1400m	$10^1 \mathrm{ms}^{-2}$	$2.\,10^{-2} \mathrm{ms}^{-2}$	$10^{-4} \mathrm{s}^{-1}$	$10^{-11} \mathrm{m}^{-1} \mathrm{s}^{-1}$	$3.\,10^6 \mathrm{m}$	$3.\,10^6 \mathrm{m}$

Mathematical Model

2-layer Shallow-water model

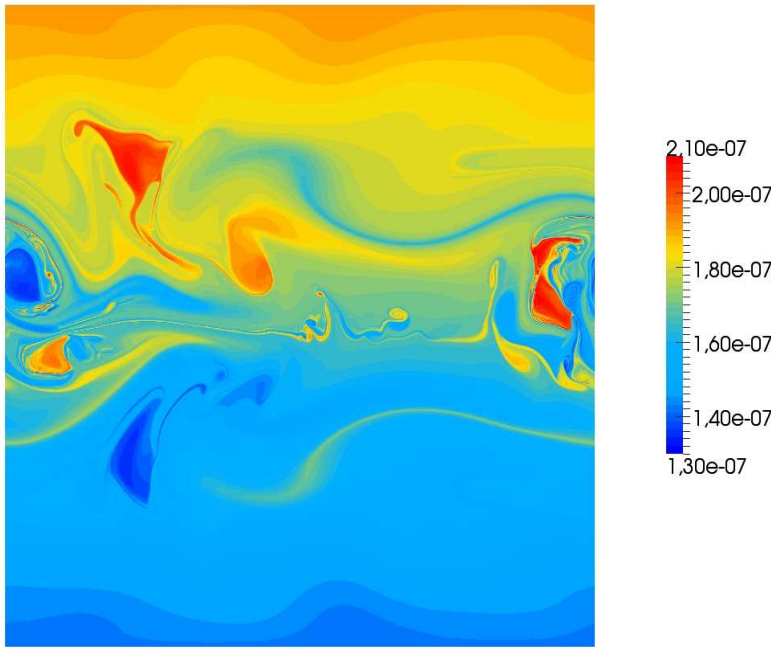
Numerical Experiment

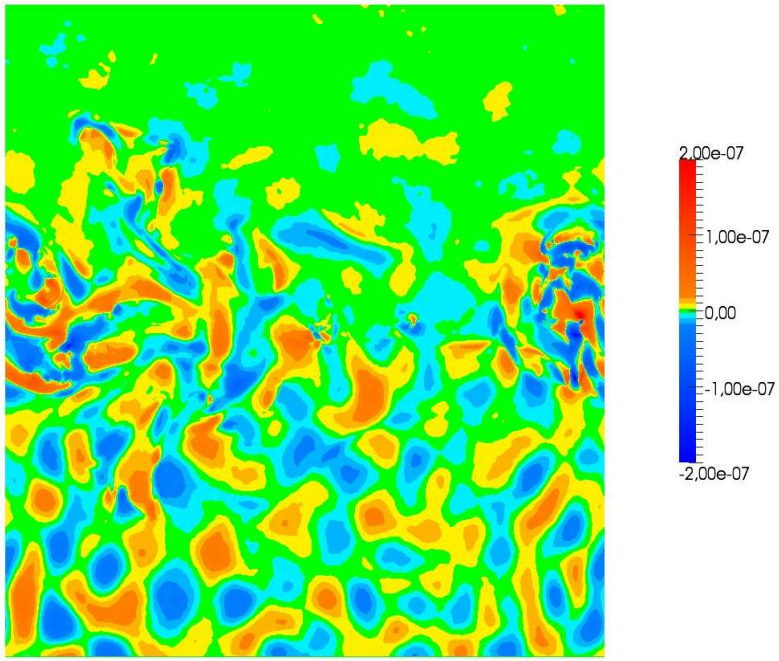
Numerical parameters : resolution 500^2 (15exp.), 2000^2 (6exp.) 4000^2 (1exp.)(750m). friction : $\nu = 1, 10, 100, 1000 \mathrm{m}^2 \mathrm{s}^{-1}$.

vortical motion vs. wave motion
potential vorticity vs. divergence

$$q = \frac{\partial_x v - \partial_y u + f_0 + \beta y}{H + \eta} \text{ vs. } d = \partial_x u + \partial_y v$$

PV vs. Div





for film see my web-page !

Divergence

$$
\begin{aligned}
\partial_t d_1 \quad &+ \quad g\nabla^2\eta_1 - f\zeta_1 = -\beta u_1 + \nu\nabla^2 d_1 \\
&- \quad \partial_x(u_1\partial_x u_1 + v_1\partial_y u_1) - \partial_y(u_1\partial_x v_1 + v_1\partial_y v_1)
\end{aligned}
$$

Source

inertia gravity wave = source

$$
\begin{aligned}
\partial_{tt} d_1 \quad - \quad & g h_1^0 \nabla^2 d_1 - g h_2^0 \nabla^2 d_2 + f^2 d_1 = \\
- \quad & \beta \partial_t u_1 + \nu \partial_t \nabla^2 d_1 + \nu f \nabla^2 \zeta_1 \\
- \quad & \partial_t \left[\partial_x (u_1 \partial_x u_1 + v_1 \partial_y u_1) + \partial_y (u_1 \partial_x v_1 + v_1 \partial_y v_1) \right] \\
+ \quad & g \nabla^2 (\partial_x ((\eta_1 - \eta_2) u_1) + \partial_y ((\eta_1 - \eta_2) v_1) \\
+ \quad & \partial_x (\eta_2 u_2) + \partial_y (\eta_2 v_2) - \kappa \nabla^2 \eta_1) \\
+ \quad & f \left[-\beta v_1 - \partial_x (u_1 \partial_x v_1 + v_1 \partial_y v_1) + \partial_y (u_1 \partial_x u_1 + v_1 \partial_y u_1) \right]
\end{aligned}
$$

Source

$$\partial_{tt} d_1 \quad - \quad g h_1^0 \nabla^2 d_1 - g h_2^0 \nabla^2 d_2 + f^2 d_1 = r_1 \tag{1}$$

if we further define the determinant and the advection operator :

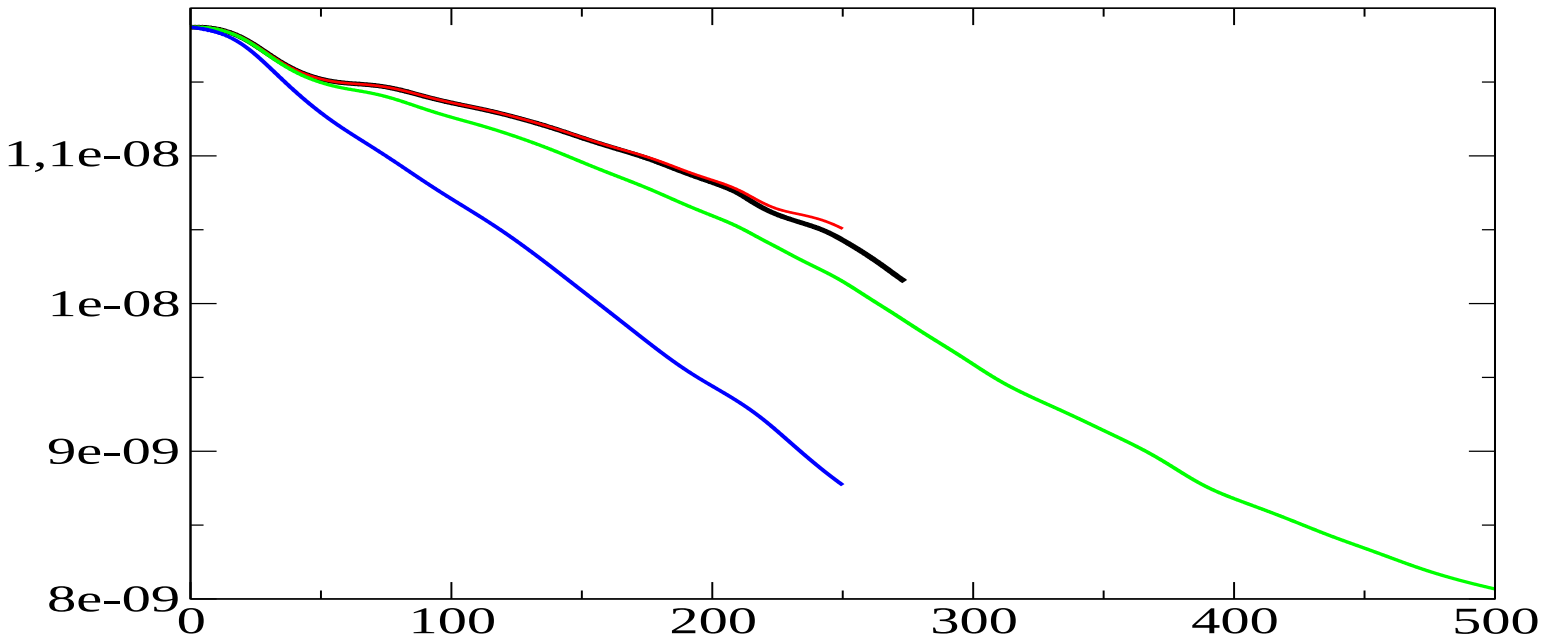
$$D_i = (\partial_x u_i)(\partial_y v_i) - (\partial_x v_i)(\partial_y u_i) \tag{2}$$

$$A_i . = u_i \partial_x . + v_i \partial_y . \tag{3}$$

we get with some reordering :

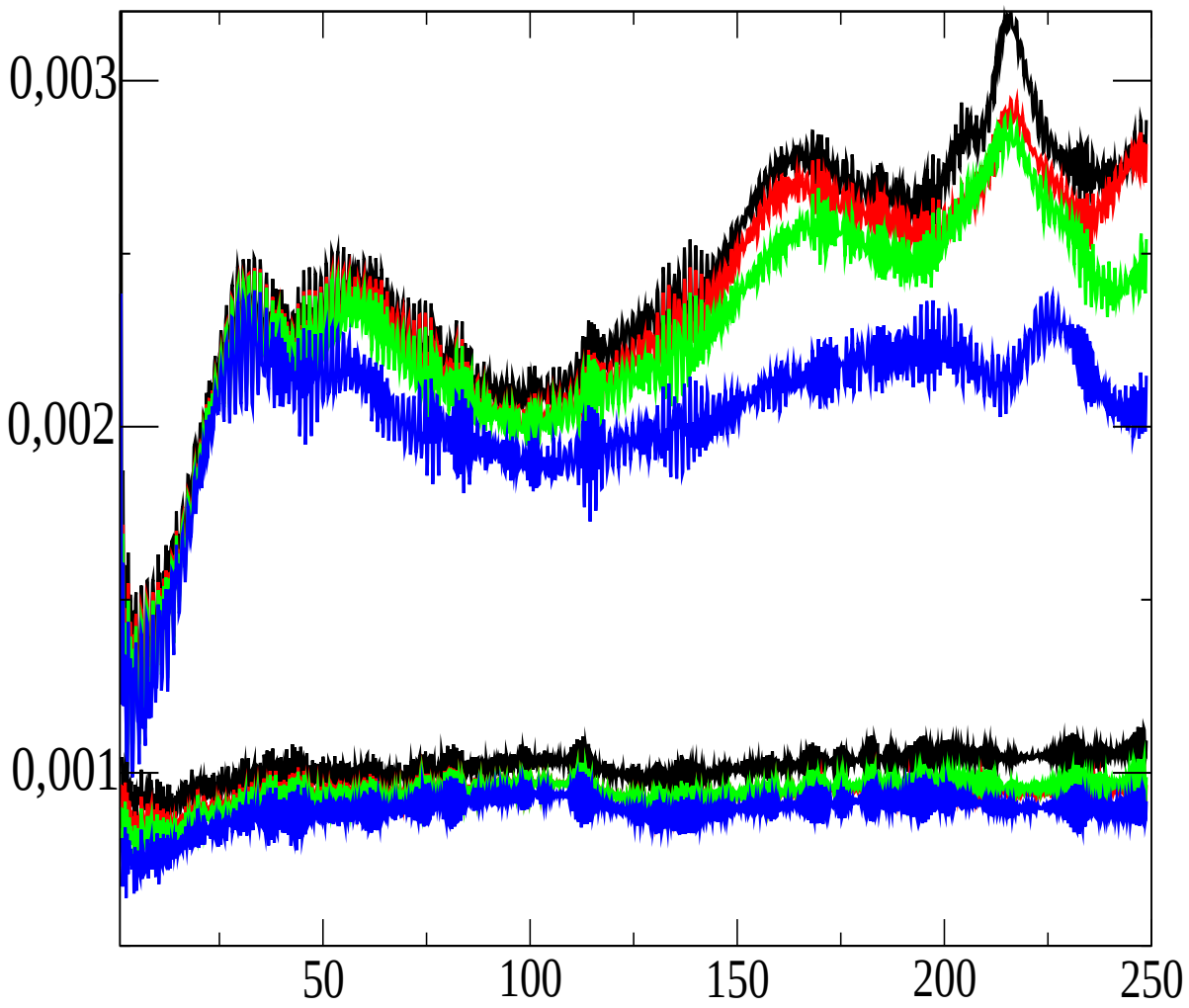
$$\begin{aligned} r_1 &= -\partial_t(\beta u_1 + A_1 d_1 + d_1^2 - 2D_1) \\ &- f(\beta v_1 + A_1 \zeta_1 + d_1 \zeta_1) \\ &+ g\nabla^2((A_1 + d_1)\eta_1 + (A_2 - A_1 + d_2 - d_1)\eta_2) \\ &+ \nu \partial_t \nabla^2 d_1 + \nu f \nabla^2 \zeta_1 - \kappa g \nabla^4 \eta_1 \end{aligned} \tag{4}$$

PV



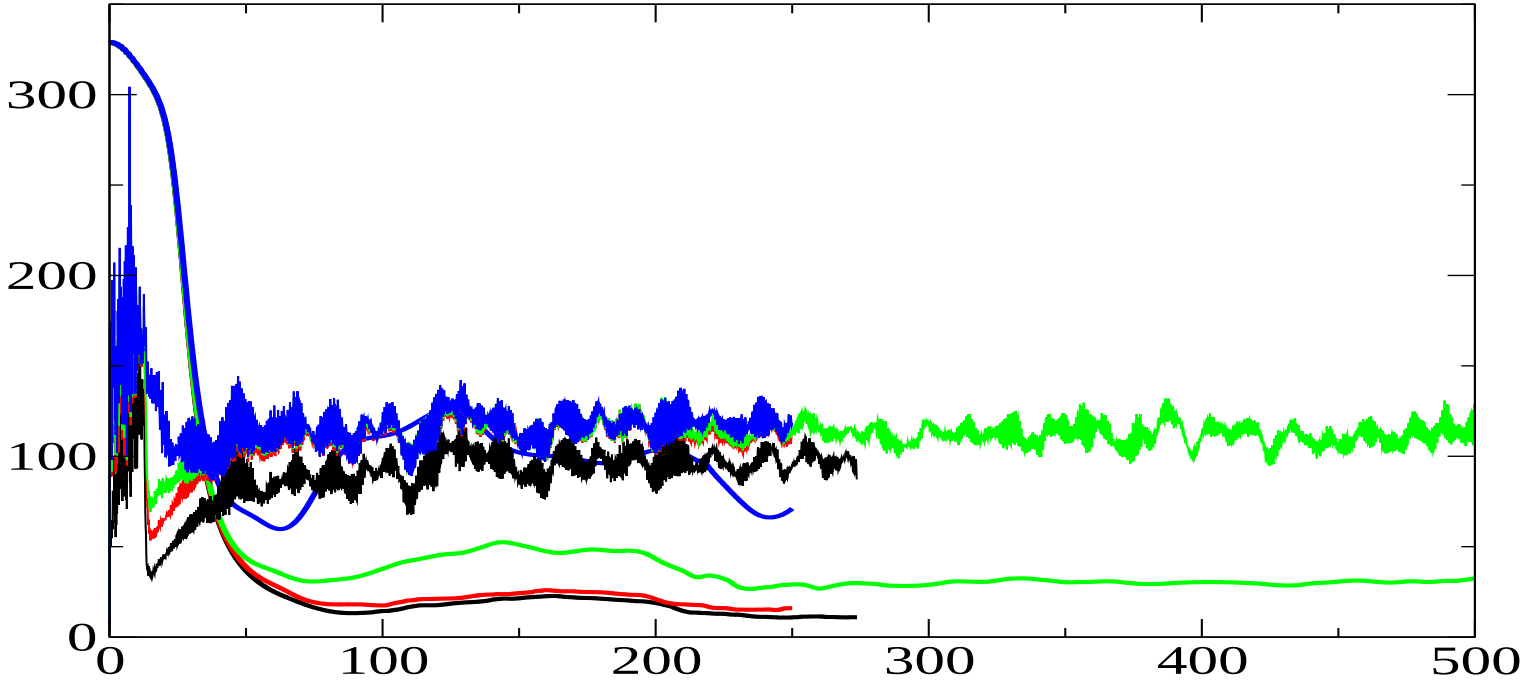
$$\bar{a_1} = \sqrt{\langle q_1^2 \rangle - \langle (\frac{f}{h_1^0})^2 \rangle}$$

Divergence

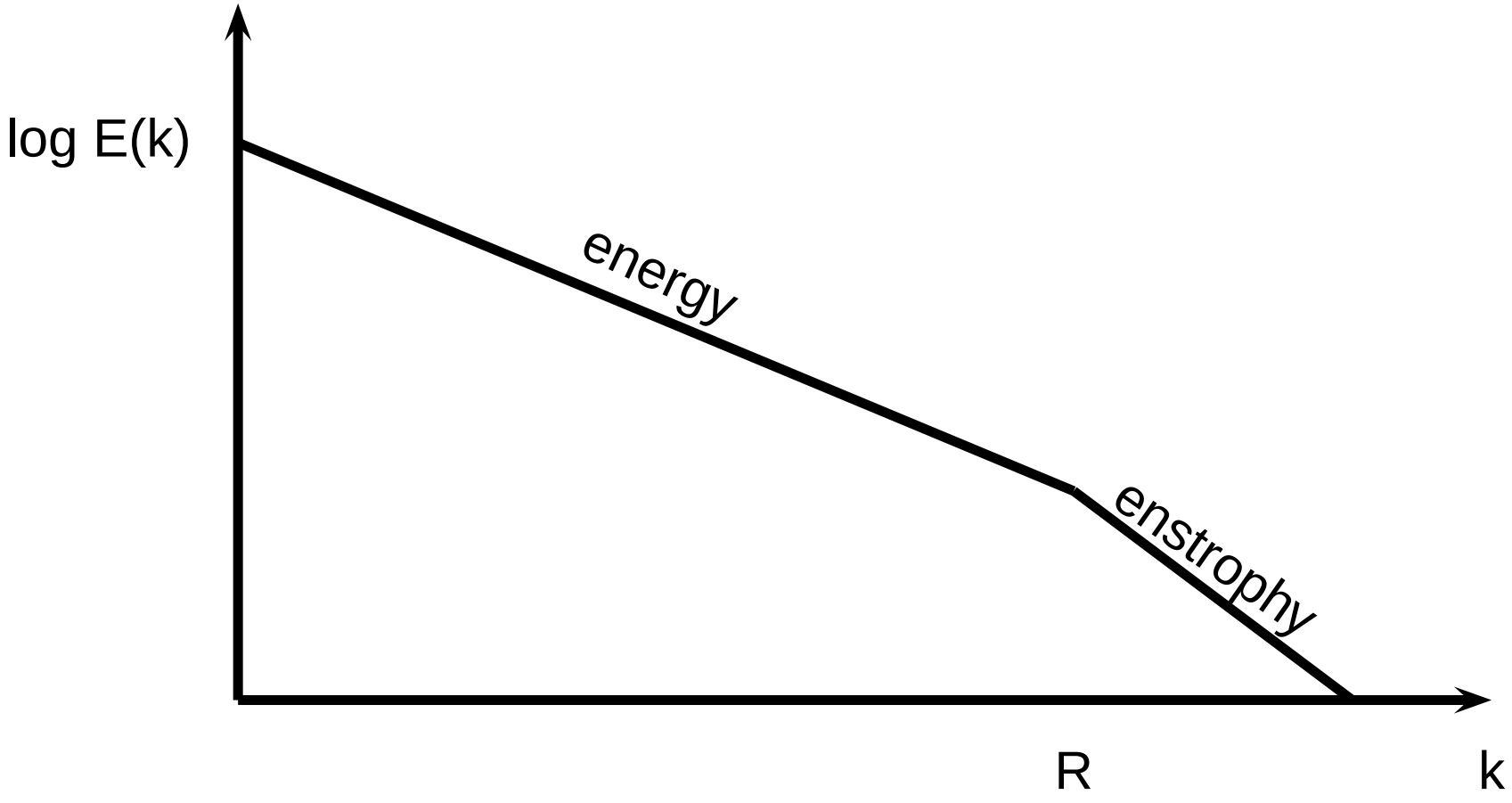


Scale

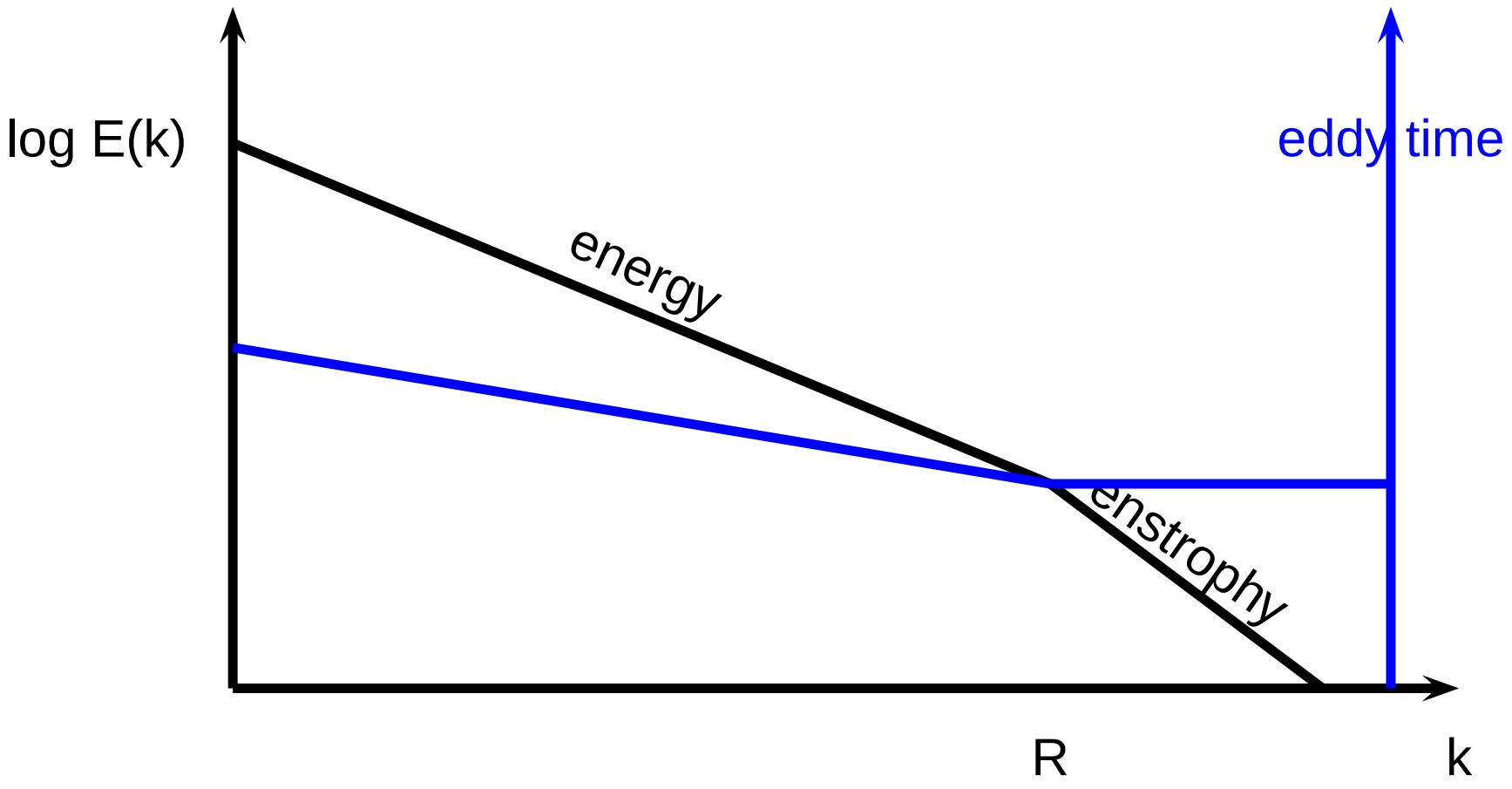
$$\frac{\langle((H\cdot(PV-f/H))^2\rangle}{E} \quad \text{and} \quad \frac{\langle d^2\rangle}{E}$$

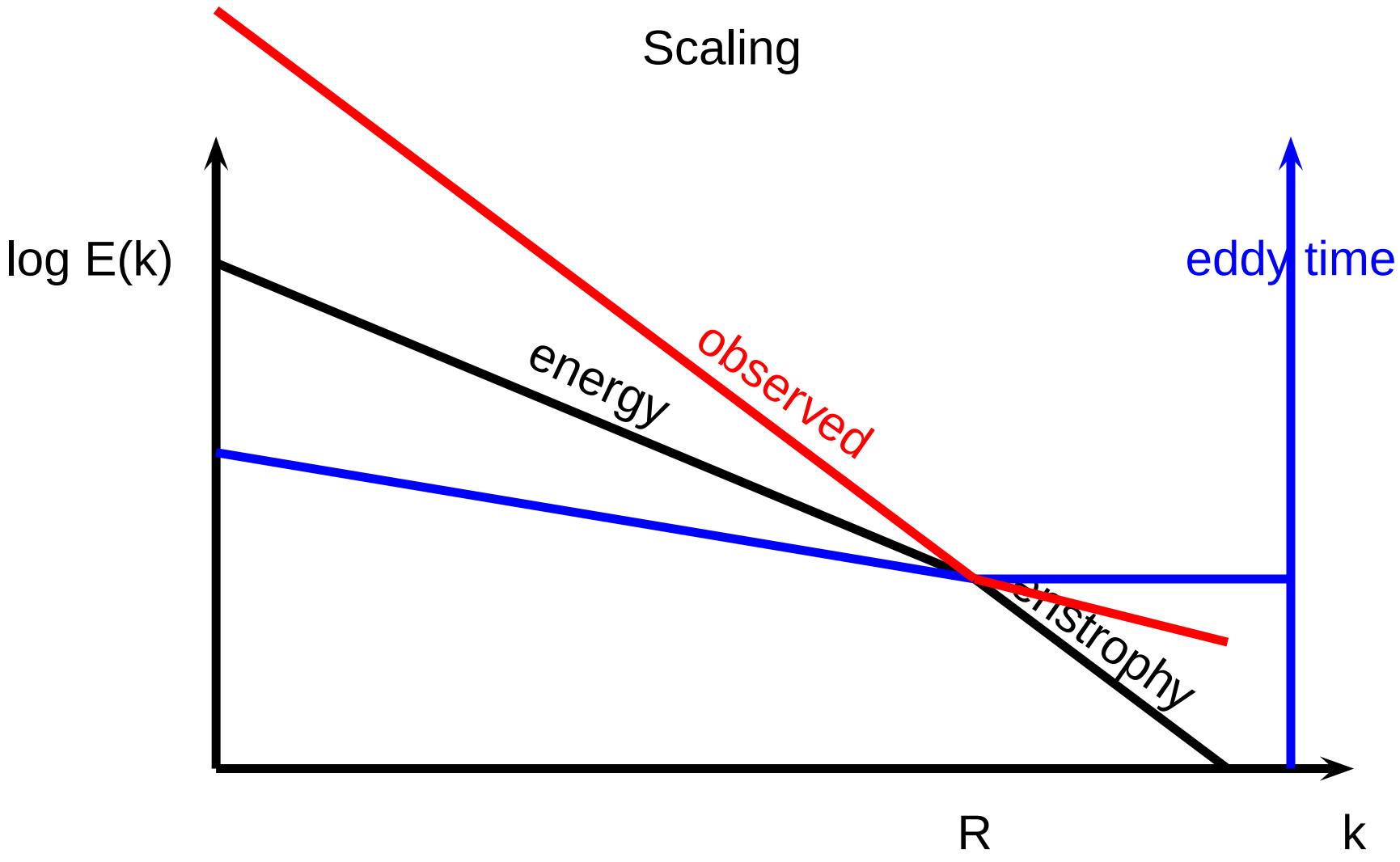


Scaling



Scaling




Scaling
log E(k)
eddy time
observed
energy
enstrophy
R
k

Conclusions

- No strong loss of balance in the enstrophy cascade (no barking) although strong fronts and eddies are observed.
- Dynamics stays close to balance even when small scales appear.
- Faint continuous gravity wave generation (growl), showing the existence of a fuzzy manifold.

Question

- How does the system manage to keep balance at small scales ? (By avoiding fast motion ?)

Perspectives

- Look at cascades of inertia-gravity wave turbulence (weak and strong).