

The Growl of near Balanced Flow in 2 Layer Shallow Water Turbulence on the β -Plane

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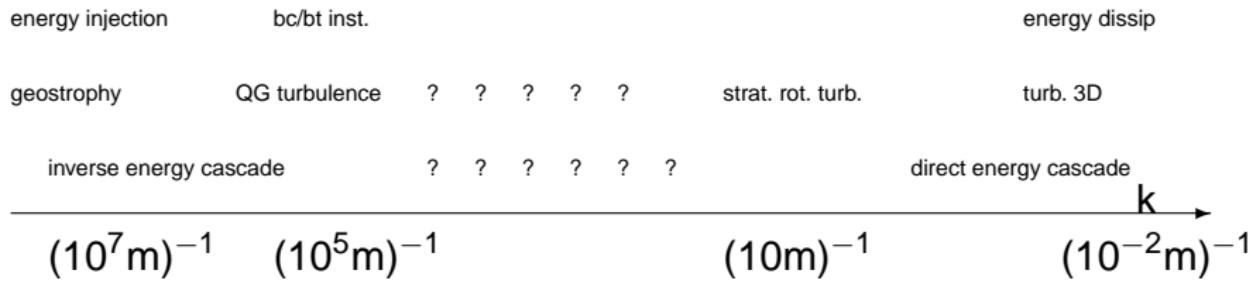
Ocean Dynamics by Scale

energy injection

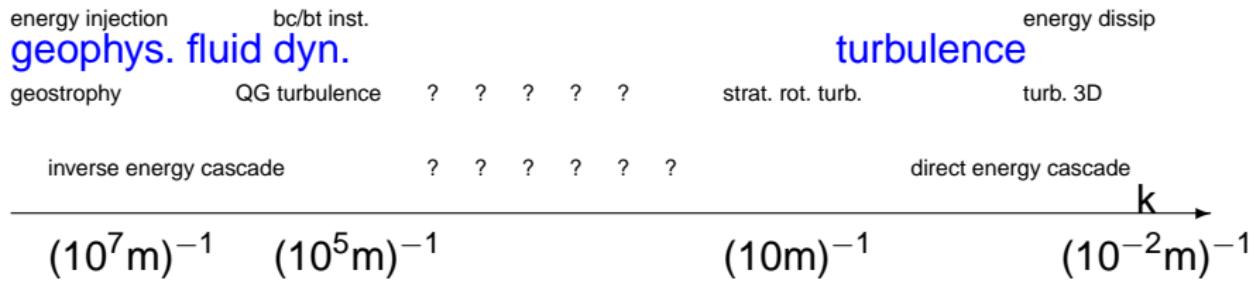
energy dissip



Ocean Dynamics by Scale

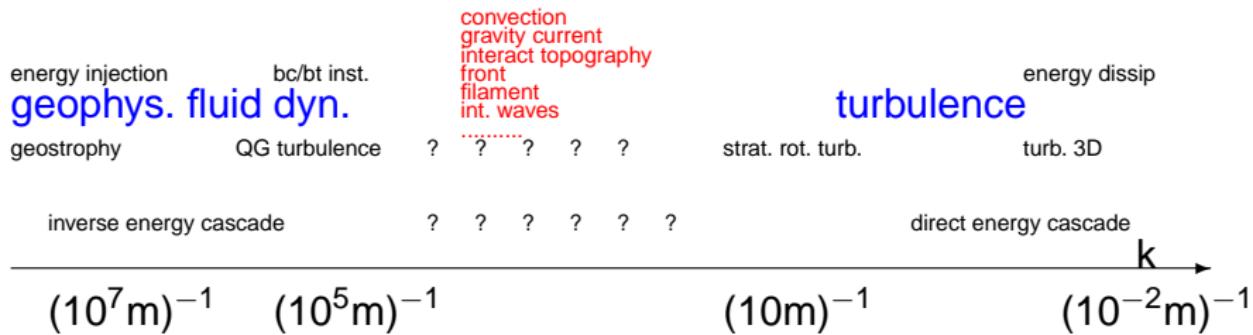


Ocean Dynamics by Scale



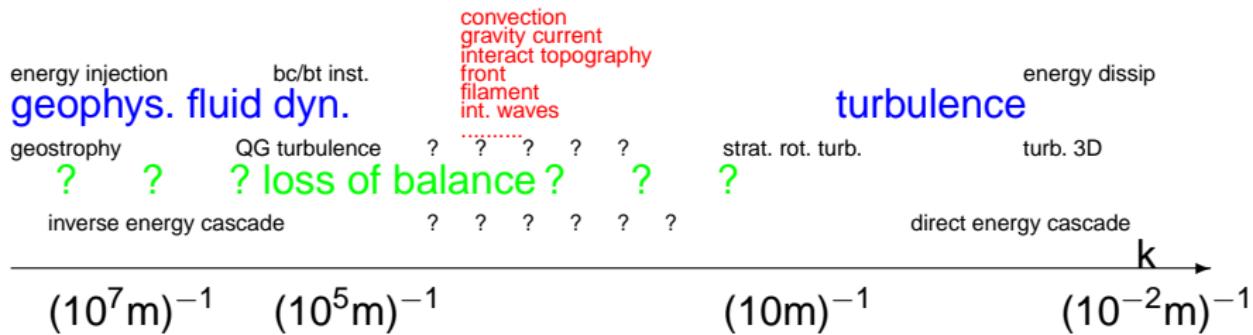
Ocean Dynamics by Scale

small scale processes



Ocean Dynamics by Scale

small scale processes



Loss of Balance $<->$ Sound Emission

Ford et al. 2000 $<->$ Lighthill 1952

Hard wave emission when :

one of criterias of balance is broken $<->$ Mach number > 1

Soft wave emission when :

???????? $<->$ due to vorticity and interaction with structures

Loss of Balance

Spontaneous (Hard) Loss of Balance (Ford et al. 2000, Molemaker & McWilliams

2006)

- ▶ unstable stratification ($N^2 < 0$)
- ▶ sign change in absolut vorticity
- ▶ sign change in the difference of absolut vorticity and horizontal strain rate
- ▶ $Fr > 1$

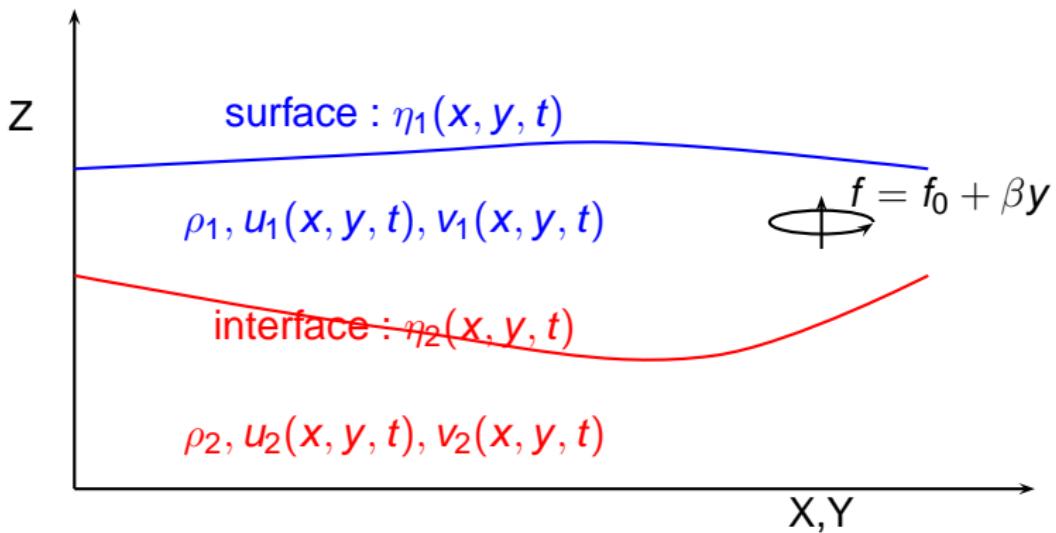
UNspontaneous (Soft) Loss of Balance

- ▶ fuzzy manifold with thickness depending on Ro and Fr .

vortical motion vs. wave motion
potential vorticity vs. divergence

$$q = \frac{\partial_x v - \partial_y u + f_0 + \beta y}{H + \eta} \text{ vs. } d = \partial_x u + \partial_y v$$

Physical model



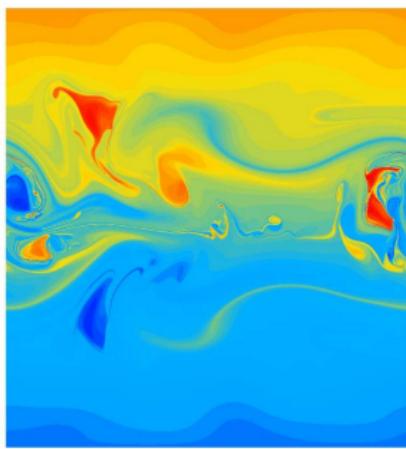
Physical Parameters

h_1	h_2	g'	g'	f	β	L_x	L_y
600m	1400m	10^1ms^{-2}	$2 \cdot 10^{-2} \text{ms}^{-2}$	10^{-4}s^{-1}	$10^{-11} \text{m}^{-1} \text{s}^{-1}$	$3 \cdot 10^6 \text{m}$	$3 \cdot 10^6 \text{m}$

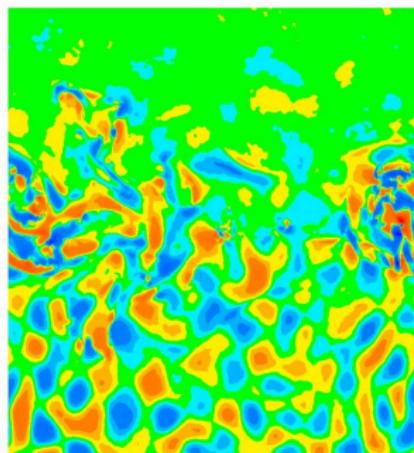
Mathematical Model 2-layer Shallow-water model Numerical Experiment

Numerical parameters : resolution 500^2 (15exp.), 2000^2 (6exp.)
 4000^2 (1exp.) (750m). friction : $\nu = 1, 10, 100, 1000 \text{m}^2 \text{s}^{-1}$.

PV vs. Div



2.10e-07
2.00e-07
1.80e-07
1.60e-07
1.40e-07
1.30e-07



2.00e-07
1.00e-07
0.00
-1.00e-07
-2.00e-07

Divergence

$$\begin{aligned}\partial_t d_1 + g \nabla^2 \eta_1 - f \zeta_1 &= -\beta u_1 + \nu \nabla^2 d_1 \\ - \partial_x(u_1 \partial_x u_1 + v_1 \partial_y u_1) - \partial_y(u_1 \partial_x v_1 + v_1 \partial_y v_1)\end{aligned}$$

Source

inertia gravity wave = source

$$\begin{aligned}\partial_{tt}d_1 - & gh_1^0 \nabla^2 d_1 - gh_2^0 \nabla^2 d_2 + f^2 d_1 = \\ - & \beta \partial_t u_1 + \nu \partial_t \nabla^2 d_1 + \nu f \nabla^2 \zeta_1 \\ - & \partial_t [\partial_x (u_1 \partial_x u_1 + v_1 \partial_y u_1) + \partial_y (u_1 \partial_x v_1 + v_1 \partial_y v_1)] \\ + & g \nabla^2 (\partial_x ((\eta_1 - \eta_2) u_1) + \partial_y ((\eta_1 - \eta_2) v_1)) \\ + & \partial_x (\eta_2 u_2) + \partial_y (\eta_2 v_2) - \kappa \nabla^2 \eta_1 \\ + & f [-\beta v_1 - \partial_x (u_1 \partial_x v_1 + v_1 \partial_y v_1) + \partial_y (u_1 \partial_x u_1 + v_1 \partial_y u_1)]\end{aligned}$$

Source

$$\partial_{tt}d_1 - gh_1^0\nabla^2d_1 - gh_2^0\nabla^2d_2 + f^2d_1 = r_1 \quad (1)$$

if we further define the determinant and the advection operator :

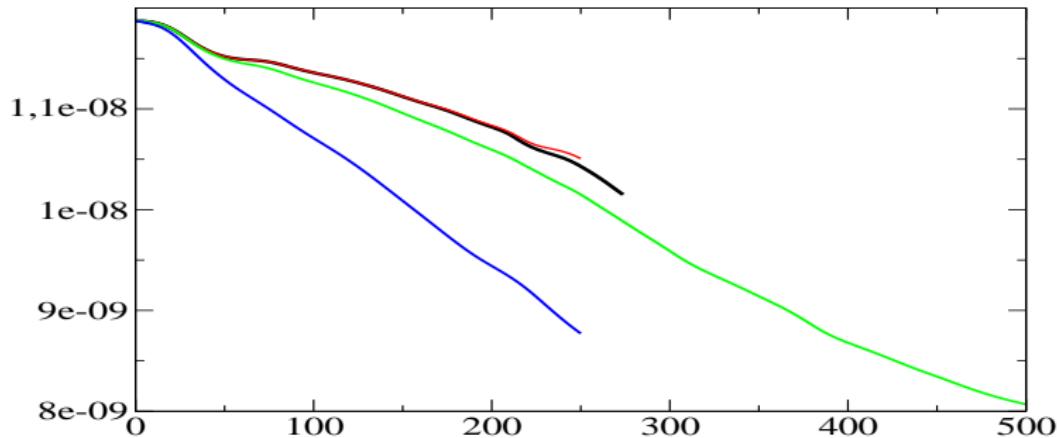
$$D_i = (\partial_x u_i)(\partial_y v_i) - (\partial_x v_i)(\partial_y u_i) \quad (2)$$

$$A_i. = u_i \partial_x. + v_i \partial_y. \quad (3)$$

we get with some reordering :

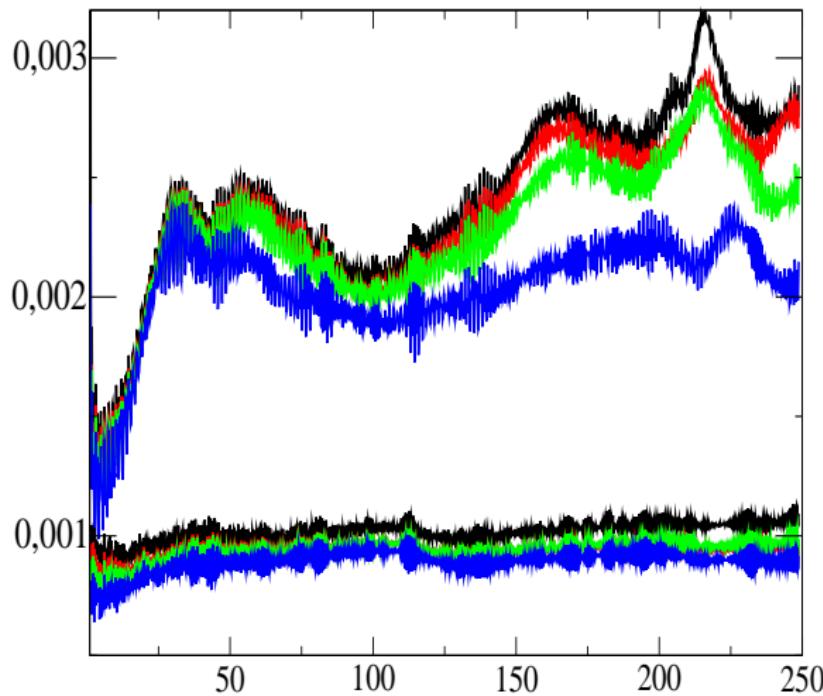
$$\begin{aligned} r_1 &= -\partial_t(\beta u_1 + A_1 d_1 + d_1^2 - 2D_1) \\ &\quad - f(\beta v_1 + A_1 \zeta_1 + d_1 \zeta_1) \\ &\quad + g \nabla^2 ((A_1 + d_1) \eta_1 + (A_2 - A_1 + d_2 - d_1) \eta_2) \\ &\quad + \nu \partial_t \nabla^2 d_1 + \nu f \nabla^2 \zeta_1 - \kappa g \nabla^4 \eta_1 \end{aligned} \quad (4)$$

PV



$$\bar{a}_1 = \sqrt{\langle q_1^2 \rangle - \langle \left(\frac{f}{h_1^0}\right)^2 \rangle}$$

Divergence



Conclusions

- ▶ No strong loss of balance in the enstrophy cascade (barking) although strong fronts and eddies are observed.
- ▶ dynamics stays close to balance even when small scales appear.
- ▶ Faint continuous sound generation (growl), showing the existence of a fuzzy manifold.

Perspectives

- ▶ Look at cascades of inertia-gravity wave turbulence (weak and strong).